ISSN 1648-4142 print / ISSN 1648-3480 online TRANSPORT
www.transport.vgtu.lt

TRANSPORT – 2007, Vol XXII, No 1, 31–37

AXLE LOAD VARIATIONS AND VEHICLE GROWTH PROJECTION MODELS FOR SAFETY ASSESSMENT OF TRANSPORTATION STRUCTURES

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Received 2 September 2006; accepted 4 December 2006

Abstract. The design formulations of pavements and bridges are significantly influenced by the uncertainties associated with the prevailing vehicle weight characteristics. Accurate modelling of load spectra and the estimation of possible vehicle growth and composition are the main components in the safety assessment of transportation structures. The vehicle weights from the existing traffic form the basis for load spectra development, and the vehicle projections are subjective to the socio-economic conditions. In this paper, the methodologies of appropriate data consideration for such representation are discussed and demonstrated based on the data from India. The load spectra are also developed for vehicular data from Interstate 95 (I-95) in New York State. In lieu of the current status of the design codes for pavements and bridges worldwide, most of the developing countries are in need of analogous models for the calibration of the design basis.

Keywords: pavement design, bridge safety, vehicle weight, axle load spectra, probabilistic modeling, nonlinear least squares, vehicle ownership, GDP per capita, population growth.

1. Introduction

The benefits of mechanistic-empirical pavement designs have been highlighted by the American Association of State Highway Transport Officials (AASHTO) [1, 2]. Methods to facilitate such designs are presently being developed by researchers worldwide [3, 4]. This involves accurate treatment of the randomness in the design variables based on observed data of vehicular characteristics. The probability distributions of such weights (load spectra) are a major requirement in the development of rational design basis of transportation structures such as pavements and bridges. The safety assessment of these structures also requires accurate simulation of vehicle weights from the load spectra and the representation of possible vehicle flow for the considered life-time, referred to as the service life of the structure [5]. Based on the available data on vehicular flow, the probable vehicle growth rates can be estimated with reference to income levels and population growth of the considered region for various classes of vehicles.

It is seen that load spectra have different characteristics depending on the vehicle class and axle position. However, the growth rates are seen to be satisfying the model requirements, independent of the vehicle type, but depending on the country [6]. There exist different choices for the selection and finalization of these models. In the present study, the characteristics and methods of such model development are discussed and the results are shown with appropriate case studies.

2. Axle weight uncertainties

The vehicle classes plying on highways consist of different types and the frequency diagrams of axle weights give an idea of the inherent uncertainties. Typical histograms of the front and tandem axle weights of a two-axle truck are shown in Figs 1 and 2. It is seen that the axle weights in some cases can be represented by conventional unimodal statistical distributions, whereas some axles indicate the presence of more than one mode as seen in Fig 2. This is attributed to the different loadings on the same class of vehicles on the highway [7]. Such trends of multimodal load spectra have been observed by many researchers [3, 8–10].

The parameters of the distribution models give an idea of the inherent uncertainties in the variables. The unimodal probability models can be arrived at using the widely used maximum likelihood approach, and the fitted models can be tested with probability plots and goodness-of-fit tests, such as the chi-square test, Kolmogorov-Smirnov (KS) test and Anderson-Darling test. Using this method, the parameters are estimated such that the likelihood of obtaining the observed database is maximized. For a set of values $\{x_1, x_2, ..., x_n\}$ with a probability density function $f(.)$, the likelihood function is defined as [11]:

$$L(x_1, x_2, ..., x_n; \theta_1, ... \theta_m) = \prod_{i=1}^{n} f(x_i; \theta_1, ..., \theta_m), \quad (1)$$

where $\theta_1, ..., \theta_m$ are the m parameters to be estimated.

The distribution parameters are obtained by differentiating eq. (1) with respect to the parameters and solving the simultaneous equations given by:

$$\frac{\partial L(x_1, x_2, ..., x_n; \theta_1, ..., \theta_m)}{\partial \theta_j} = 0; \quad j = 1, ..., m. \quad (2)$$

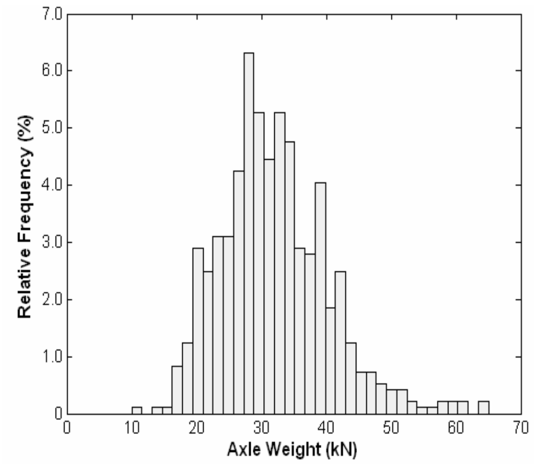


Fig 1. Frequency diagram for single axle weight

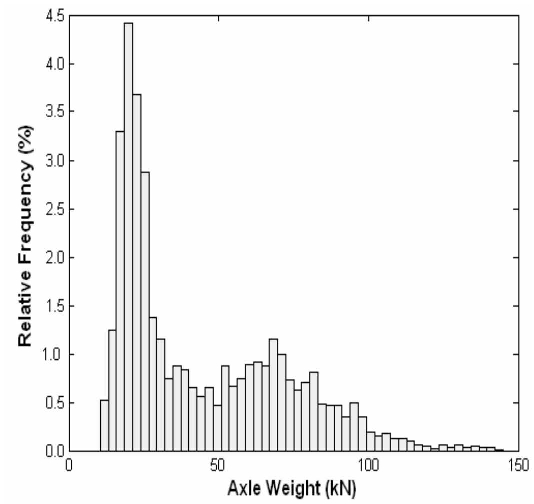


Fig 2. Frequency diagram for tandem axle weight

The axle weights with multi-modes are complex and are required to be considered appropriately as per the proportion of segments. Fig 3 shows the Cumulative Distribution Function (CDF) of the empirical distribution and a unimodal distribution for the multimodal data. It is seen that the individual statistical distributions cannot represent the models in these cases. The earlier attempts to consider these models involved the separation of axle weight data based on the permissible limits; however, these models did not pass the required statistical tests [8]. Further studies on developing load spectra included the polynomial regression analysis of the smaller segments of the frequency diagrams [3, 9]. In this approach, the model requirements are satisfied in lieu of the smaller segments. However, in cases of lesser amounts of vehicular database, the practical use of such models is very limited.

A much better approach to consider the multimodal distributions is by involving a mixture of probability distributions for load characterization. These models are very useful in a wide range of practical and theoretical applications and can be checked with the regular hypothesis tests. The presence of multi-modes requires the statistical model to be representing it correctly.

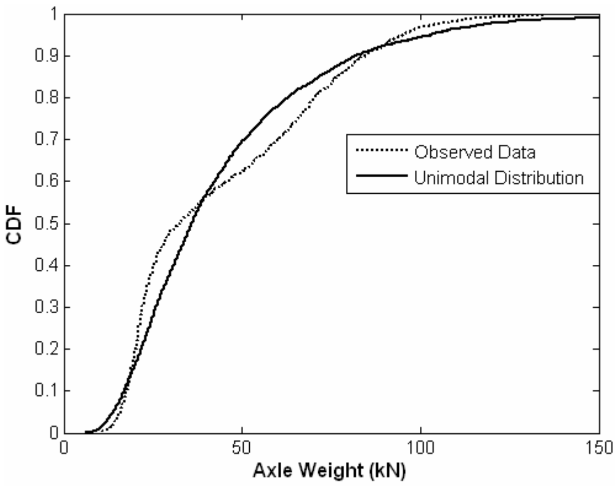


Fig 3. Observed and empirical distributions

Estimation of multimodal load spectra

The multimodal distribution model ($\hat{f}(w)$) is derived by expressing it as the weighted sum of univariate marginal distributions ($f_i(w)$) defined as:

$$\hat{f}(w) = \sum_{i=1}^{n} p_i f_i(w), \quad (3)$$

where p_i is the proportion of the i-th univariate model.

Various marginal distributions and their combinations can be tried for the available vehicle weight database and the best fitting distribution models can be obtained.

In the present study, the parameters of the distributions and the corresponding proportions are obtained using the nonlinear least squares fitting of the observed data. The trust-region algorithm is used for the nonlinear fitting as it is more efficient compared to other popular approaches [12]. In this approach, an approximate and simpler function is chosen to realistically represent the original function that is to be optimised. The approximate model is trusted in the neighbourhood of the current iterate and the region is recomputed with the iterations. If the model representation is acceptable, the trust region is expanded otherwise it is contracted. The Taylor series representation of the original function to the second degree at the considered point is taken as the approximating model.

The trust-region algorithms can use non-convex approximate models, are reliable and robust, and have excellent convergence properties [13]. The trial step computation at the i-th iteration [14, 15] is the solution of

$$\min\left[g_i \mathbf{k}^T + \frac{\mathbf{k}^T \mathbf{H}_i \mathbf{k}}{2} \right]; \ \mathbf{k} \in \Re$$
$$s.t. \ \left\| \mathbf{Dk} \right\| \le \Delta_i, \qquad (4)$$

where g_i is the gradient of the original function at the current point, **k** is the trial step, **H** is the symmetric matrix approximating the Hessian, **D** is a diagonal scaling matrix, $\Re$ is the set of Real numbers and $\Delta_i > 0$ is a trust region radius. In each iteration, the ratio of the actual and predicted reduction of the objective function is the criteria for acceptance of the point. The selection of final distribution model is based on the value of the coefficient of determination and on the two sample KS test at 2 % significance level.

Load spectra

The methodologies described in the earlier sections are demonstrated by applying to the vehicular data obtained from highways around New Delhi and from I-95 in New York. Being the capital of India, and with heavy industrialization around the city, New Delhi is expected to have the heaviest of axle weights with different types of trucks. The Interstate highway 95 runs along the east coast of the United States and is one of the most heavily travelled highways. The data from these locations consists of a variety of vehicular cases and suits the objective of the present study.

The load spectra are derived by applying the appropriate models to vehicular data. The marginal statistical distributions are chosen based on the value of coefficient of determination and the hypothesis test results as described earlier. The rear, middle and tandem axle weights are found to be best described with the mixed distributions having the corresponding inverse Gaussian (IG), lognormal (LN) and normal (N) distributions. A typical frequency diagram and the selected distribution model are shown in Fig 4 and a typical CDF plot of the fitted model is shown in Fig 5. It is seen that the observed data is very closely approximated by the fitted model of the spectra.

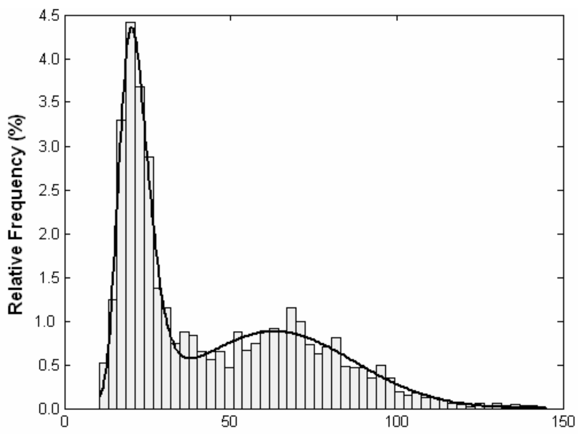


Fig 4. Typical axle weight distribution

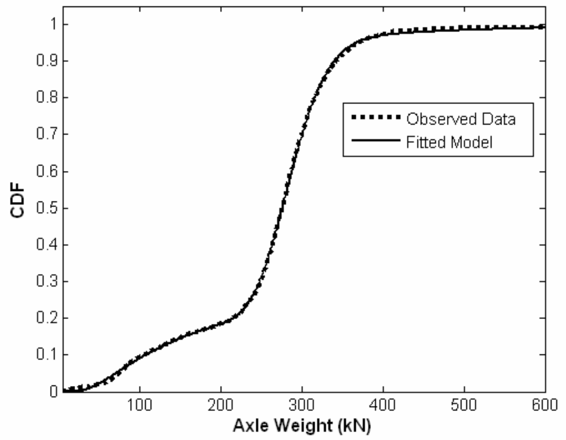


Fig 5. Typical CDF plots of observed and fitted data

The selected distribution models and the statistical descriptions of the typical vehicle cases at the two locations New Delhi (ND) and New York (NY) are given in Table 1. The case of third mode is also observed for a vehicle at NY. The load spectra thus obtained will be combined with the extrapolation models described in the next section to carry out the load characterization and safety assessment studies.

3. Vehicle growth projections

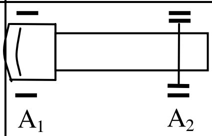
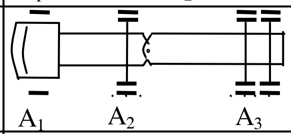
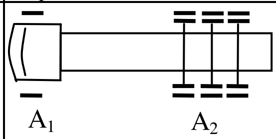
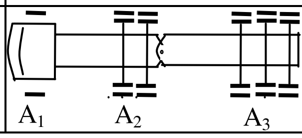
In assessing the safety of the transportation structures, it is required to consider the possible vehicle growth in the considered service period. The vehicle ownership in any country depends on the existing social conditions and the level of economic growth [6, 16, 17]. The population increase rate and the Gross Domestic Product (GDP) are the significant factors influencing the vehicle population models.

It is observed that representative models can be developed by relating the vehicle ownership with the GDP per capita of corresponding years for different vehicle types. Suitable functional forms are required to be derived to represent the relationship between the vehicle ownership and per-capita income. The standard approach to represent these relations is by applying the sigmoid functions. There exist many choices for choosing the suitable functions such as the quassi-logistic, logarithmic logistic, cumulative normal, and Gompertz functions. The vehicle ownership models also assist in estimating the income elasticities of the vehicle classes.

The significant studies involving the road vehicle growth are based on the models proposed by Dargay [6] and Button [16]. It is seen in general that the increase of ownership levels of vehicles involve three stages of growth [17]:

- a slow growth rate at lower income levels;
- a rapid growth rate at increased levels;
- a slow growth rate as saturation levels of vehicles are approached.

Table 1. Probabilistic descriptions of typical vehicle axle weights

Location	Vehicle Description	Axle	First mode				Second mode				Third mode			
			p_1	f_1	E^a (kN)	COV^b	p_2	f_2	E (kN)	COV	p_3	f_3	E (kN)	COV
ND		A_1	–	IG	19.93	0.35	–	–	–	–	–	–	–	–
		A_2	0.55	LN	22.46	0.26	0.45	IG	70.15	0.30	–	–	–	–
ND		A_1	–	IG	22.50	0.26	–	–	–	–	–	–	–	–
		A_2	0.49	IG	24.09	0.35	0.51	LN	59.70	0.21	–	–	–	–
		A_3	0.43	IG	43.41	0.29	0.57	LN	116.88	0.25	–	–	–	–
NY		A_1	0.69	LN	70.06	0.22	0.31	N	33.38	0.39	–	–	–	–
		A_2	0.28	IG	194.6	0.36	0.72	LN	288.28	0.13	–	–	–	–
NY		A_1	0.65	LN	49.5	0.19	0.35	IG	35.39	0.24	–	–	–	–
		A_2	0.56	IG	122.60	0.38	0.44	LN	181.41	0.20	–	–	–	–
		A_3	0.57	LN	142.8	0.34	0.40	N	275.98	0.27	0.03	IG	149.90	0.12

[a] E is the mean value of the corresponding axle weight
[b] COV is the coefficient of variation

Button [16] observed that over an extended period of time, countries with lower GDP levels will follow the trends made by the industrialized nations. This factor is considered by introducing a non-dimensional "time trend" (T) in the models with $T = 1$ for the starting year with the available data. The sensitivity of the model to the base year was checked by using the quassi-logistic and the log-linear forms. The saturation level (S), income level (G), country specific variable (c_k) and time trend (T) are incorporated in the following model with the vehicle ownership as:

$$V = \frac{S}{1 + e^{(-a - \sum \delta_k c_k)} G^{-b} T^{-t}}, \tag{5}$$

where a, b and t are the constants to be estimated. The same data is used to formulate the log-linear model with constants and is given by:

$$V = \exp(a + \sum \delta_k c_k) G^b \exp(Tt). \tag{6}$$

The low income countries were classified into five groups based on the per capita income, and ownership characteristics and the modelling statistics are based on 1980 prices.

Dargay and Gately [17] prescribed using the Gompertz equation for estimating the vehicle growth over the considered period. The variations over the period of 1960–1992 were considered for model estimations. The relation between vehicle ownership and the GDP per capita is given by the Gompertz model of the form:

$$V^* = \gamma_1 \exp\left(\alpha_1 \exp\left(\beta_1 G\right)\right), \tag{7}$$

where γ_1 is the saturation level, and α_1 and β_1 are the negative parameters defining the shape or curvature of the function. The lags in the vehicle ownership and per capita income were considered by partially adjusting the income estimates. To consider the characteristics of the parameters of the Gompertz function, the shape parameters were assumed to be same for all countries.

The parameter β_1 is allowed to be country-specific.

The approximation of the model involves the following two steps:

- formulation of the database based on the levels of population growth and the economic status from the past decades;
- studying the plot of vehicle ownership with the GDP per capita and choosing the appropriate sigmoid function.

These models are closely dependent on the country of consideration and a typical application to the data from India is described in the next section. The vehicles are in general classified as cars, buses and commercial vehicles. The proportion of different commercial vehicle types can be extracted from the overall percentage of that class. The development of models elsewhere can be carried out with the same approach.

Application to Indian scenario

The extensive data available from various publications of the Central Statistical Organisation of the Ministry of Statistics & Programme Implementation, Government of India, is used as the modal basis [18, 19, 20]. The database of different vehicle classes registered in India is available from the year 1951. The initial years 1951–1991 have a discrete yearly observed database, with extensive yearly data afterwards. The data of vehicle classes is analyzed with respect to the corresponding yearly population and the variation is shown in Fig 6. The GDP characteristics are available extensively from 1951 for every year and the per capita variation is studied to formulate the model. The observed database is depicted in Fig 7.

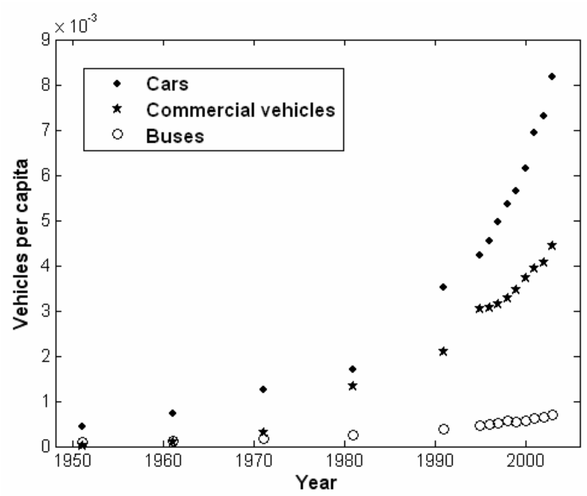


Fig 6. Variation of yearly vehicles per capita

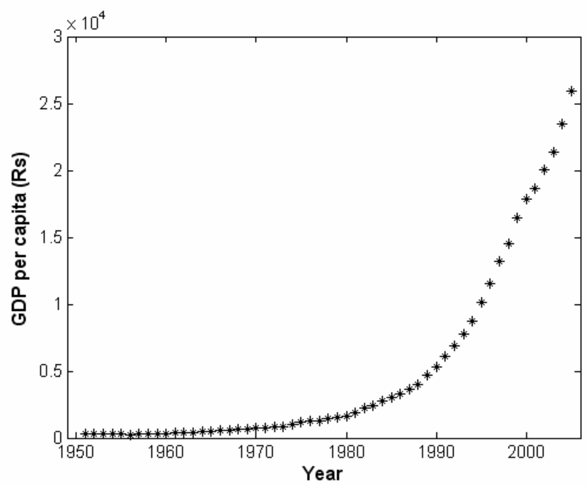


Fig 7. Variation of yearly GDP per capita

The yearly variation of vehicle ownership and GDP per capita from Figs 6 and 7 are rationally combined to study the models, and the observed variation of these factors for different vehicle types is shown in Fig 8. It is seen that the vehicle ownership levels are in the second stage of rapid growth rate due to increased income levels as outlined earlier.

The models specified in equations (5), (6) and (7) are tested for obtaining the parameters more accurately using the large databases presently available in India. After carrying out many trials, the following sigmoid function is found to be most appropriate to express the vehicle ownership representing the database as:

$$V = \exp(a) G^{b} T^{t}, \quad (8)$$

where a, b and t are the constants obtained from nonlinear regression analysis using least squares estimation. The available database is considered to obtain the parameters of the above equation and their values are given in Table 2. The estimates of the fitted model for the vehicle classes are shown in Fig 9. It is seen that the models approximate the available data closely.

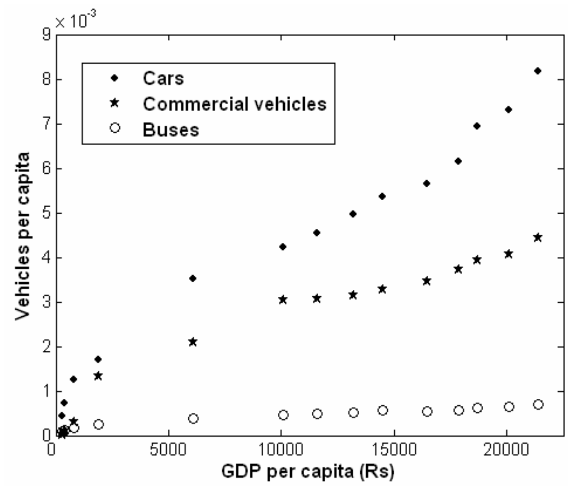
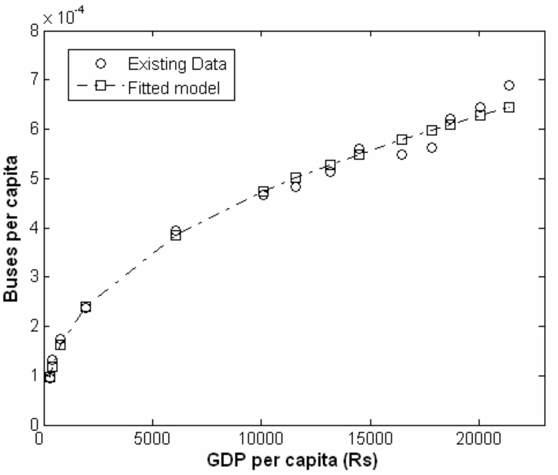


Fig 8. Variation of yearly vehicle ownership and GDP per capita

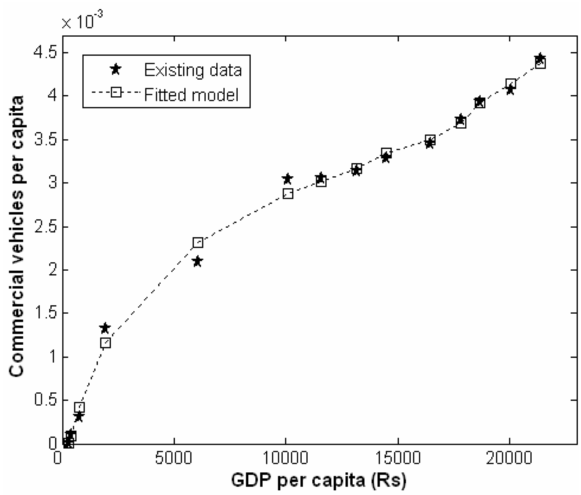
Table 2. Parameters of the model

Vehicle Class	a	b	t
Cars	–11.579	0.5814	0.4777
Commercial vehicles	–10.974	0.2272	4.2801
Buses	–11.454	0.3947	0.0950

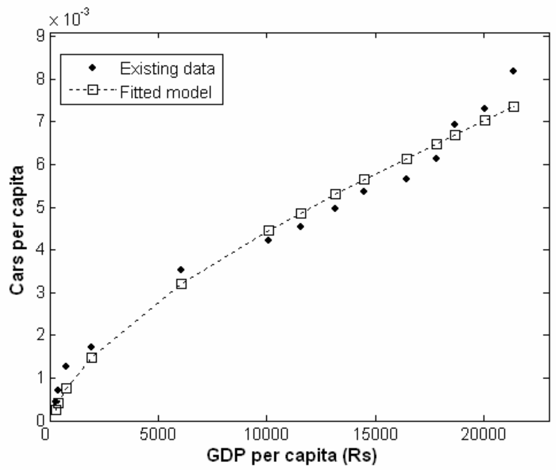
The expected vehicles per capita estimates corresponding to the future years require the probable GDP per capita growth. The GDP growth characteristics for Indian conditions are considered from the World energy outlook 2002 of the International energy agency. These values are used in equation (8) to estimate the vehicle ownership values for the various classes, and the projected values of the yearly vehicle ownership are shown in Fig. 10. It is seen that the possible number of buses per capita remain almost the same while the commercial vehicle ownership may exceed that of cars after a period of time. The projected values of population are used to estimate the probable number of vehicles for the future years and the obtained values along with the available database are shown in Fig 11. These models will also be helpful in the assessment of possible energy use and emission characteristics from the vehicles.



a) Variation of buses per capita



b) Variation of commercial vehicles per capita



c) Variation of cars per capita

Fig 9. Estimates of the fitted model

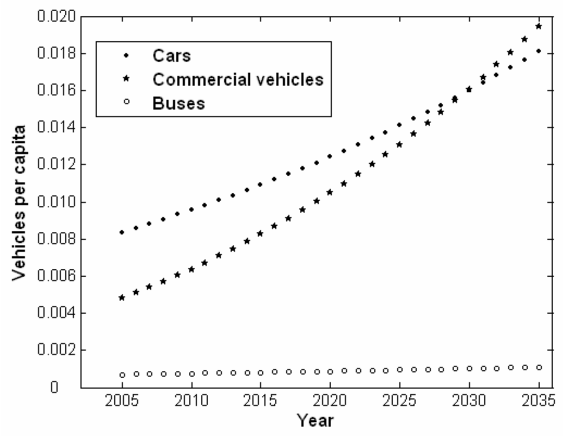


Fig 10. Projected vehicle ownership values

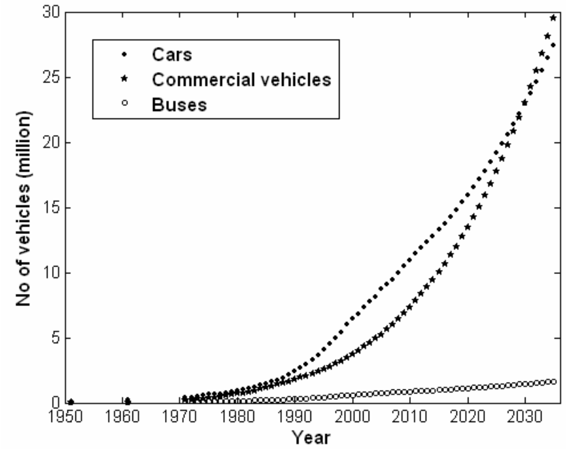


Fig 11. Observed and projected number of vehicles

4. Conclusions

The importance of vehicle weight uncertainties and the possible vehicle growth in the performance based design specifications of transportation structures are highlighted. The load spectra of vehicle weights involve representation of multiple modes representing various levels of loading, and the methodologies to estimate the accurate spectrum are developed based on the nonlinear least squares approach. Typical cases of vehicular weights from New Delhi and New York comprising different traffic conditions are illustrated. It is seen that the trust-region algorithm represents the nonlinear models closely. The basis for the number of vehicle passages with the composition expected during the service lives of these structures are formulated to be based on the existing socio-economic characteristics. The sigmoid function models suitable for such studies are discussed and are shown with the extensive data on the yearly variation of vehicle ownership and the income levels obtained from the Central Statistical Organisation, Government of India. It is seen that the ownership levels of commercial vehicles and cars change significantly compared to the buses. The commercial vehicle proportion includes the various trucks plying on the roads and can be considered accordingly.

The models presented in this paper play an important role in the present scenario of code calibration studies worldwide and can be adopted with the prevailing database.

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