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DUAL HESITANT FUZZY AGGREGATION OPERATORS

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Abstract. Dual hesitant fuzzy sets (DHFSs) is a generalization of fuzzy sets (FSs) and it is typical of membership and non-membership degrees described by some discrete numerical. In this article we chiefly concerned with introducing the aggregation operators for aggregating dual hesitant fuzzy elements (DHFEs), including the dual hesitant fuzzy arithmetic mean and geometric mean. We laid emphasis on discussion of properties of newly introduced operators, and give a numerical example to describe the function of them. Finally, we used the proposed operators to select human resources outsourcing suppliers in a dual hesitant fuzzy environment.

Keywords: DHFSs, dual hesitant fuzzy arithmetic mean, dual hesitant fuzzy geometric mean, aggregation operator, human resources outsourcing suppliers.

JEL Classification: C02, C44, D81, O15.

Introduction

Fuzzy set (FS) theory (Zadeh 1965) is a powerful technique for depicting indefiniteness. In order to give a more detailed description of an uncertain world, many extended forms of FS theory have been proposed. For example, Zadeh (1975) extended FSs and erected the theory interval-valued FSs (IVFSs). Years later, the type-2 fuzzy set was proposed by Dubois and Prade (1980). Yager (1986) introduced the fuzzy multiset as another generalization of FS. An intuitionistic FS (Atanassov 1986) has three main parts: a membership, non-membership and hesitancy (Xu 2007). Torra and Narukawa (2009) and Torra (2010) proposed another generalization of FS – the hesitant fuzzy set (HFS) – that allows the membership degree descried by a set of discrete numerical (Zhang, Xu 2015; Yu 2014a; Yu *et al.* 2013; Xia *et al.* 2013; Wei 2012; Xia, Xu 2011).

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Dual hesitant fuzzy sets (DHFS) are a generalization of FS first proposed by Zhu *et al.* (2012a). These are characterized by membership and non-membership degrees that are represented by sets of possible values (Ye 2014; Yu 2014b). DHFS is an efficient mathematical approach for studying imprecise, uncertain, or incomplete information or knowledge. It is an invaluable aid in cases where there are troubles in establishing the membership and non-membership of an element belongs to a set (Xia, Xu 2011). For example, three reviewers want to estimate the degrees to which a candidate satisfies the criterion of honesty. Because they have never seen each other before, the entire evaluation process is conducted in an uncertain environment. The first reviewer thinks that the degree of honesty for this candidate is 0.6, and that s/he has a 0.3 possibility of being dishonest. Meanwhile, the second reviewer regards that the degree of honesty is 0.7, and in his opinion, this candidate only has 0.2 possibility to be a dishonest man. Similarly, the third reviewer believes that the possibility of honesty is 0.5 while the contrary is 0.1. We assume that the above three reviewers have the same degree of influence on the evaluation and that there is no mutual interference among them. In this circumstance, the integrated information of the candidate's honesty can be expressed as a dual hesitant fuzzy element (DHFE) $\{\{0.5,0.6,0.7\},\{0.1,0.2,0.3\}\}$. Another example, the review of a PhD thesis in China is always anonymously taken by three experts, this determines that those experts have no way to exchange ideas. Due to the complexity of reviewing a PhD thesis, it is very difficult for an expert to provide accurate evaluating values. The first expert thinks that the possibility of the PhD thesis meeting the requirements is 0.7 and that of it not being up to the standard is 0.3. The second one believes that the chance that the PhD thesis meets the requirements is 0.6 while the contrary is 0.2. The third expert regards the compliance to be 0.5 and the non-compliance to be 0.3. In these situations, the degree to which the PhD thesis meets the requirements can be expressed as a DHFE $\{\{0.5,0.6,0.7\},\{0.2,0.3\}\}$. If we use a hesitant fuzzy element to represent this situation, the result is $\{0.5,0.6,0.7\}$. We found that the hesitant fuzzy element $\{0.5,0.6,0.7\}$ only expresses the membership degree but completely ignores the non-membership degree to which the PhD thesis meets the requirements. Therefore, it is far better to represent the situation by using a DHFE than a hesitant fuzzy element.

Information aggregation is one of the fields to which FS theory and extended FS theories have been applied extensively (Yager, Kacprzyk 1997; Calvo *et al.* 2002; Torra 2003; Xu, Da 2003; Bustince *et al.* 2007; Li 2010; Wei 2010; Fernando Umberto 2013; Kosareva, Krylovas 2013; Zhao, Wei 2013; Zhang 2013; Zhu *et al.* 2012b; Xu 2005, 2007, 2010, 2011; Yu 2015). However, there seem to have been no investigations on dual hesitant fuzzy information aggregation. This article aims at investigating aggregation methods for DHFEs. To achieve this target, we arranged the rest of this paper as follows. Section 1 reviews some fundamental theory about DHFS briefly. Section 2 develops the dual hesitant fuzzy weighted averaging (DHFWA) and dual hesitant fuzzy weighted geometric (DHFWG) operators, the desirable properties of which are also investigated in this section. Section 3 examines problems involving the selection of human resources outsourcing suppliers based on the proposed operators. The last Section carries on the summary to the whole paper.

1. Preliminaries

As a generalization of FS, the HFS was first put forwarded by Torra and Narukawa (2009).

Definition 1 (Torra, Narukawa 2009; Xia, Xu 2011). Suppose there is an objective set and marked by X, an HFS is defined as follows:

$$E = \{< x, h_E(x) > | \, x \in X\}, \tag{1}$$

$h_E(x)$ in Eq. (1) is a real numbers set belongs to [0,1] and it shows the membership degree of the basic element $x \in X$.

Zhu *et al.* (2012a) proposed another generalization of an FS called DHFS.

Definition 2 (Zhu *et al.* 2012a). Suppose there is an objective set and marked by$X$. A DHFS D is defined as:

$$D = \left\{ \left\langle x, h(x), g(x) \right\rangle x \in X \right\}, \tag{2}$$

$h(x)$ and $g(x)$ in Eq. (1) are two real numbers set belongs to [0,1] and they convey the membership degree and non-membership degree of the basic element $x \in X$. Furthermore,

$$0 \le \gamma, \eta \le 1, \; 0 \le \gamma^+ + \eta^+ \le 1, \tag{3}$$

where $\gamma \in h(x), \eta \in g(x)$, and for any $x \in X$, $\gamma^+ \in h^+(x) = \bigcup_{\gamma \in h(x)} \max\{r\}$ and $\eta^+ \in g^+(x) = \bigcup_{\eta \in g(x)} \max\{\eta\}$. We know from the concept of HFS (Torra, Narukawa 2009; Torra 2010) that $h(x)$ and $g(x)$ are two HFSs.

For convenience, Zhu *et al.* (2012a) defined the two dimensional arrays $d(x) = \left(h(x), g(x)\right)$ as a DHFE, denoted by $d = \left(h, g\right)$, with the conditions $\gamma \in h$, $\eta \in g$, $\gamma^+ \in h^+ = \bigcup_{\gamma \in h} \max\{r\}$, $\eta^+ \in g^+ = \bigcup_{\eta \in g} \max\{\eta\}$, $0 \le \gamma, \eta \le 1$, and $0 \le \gamma^+ + \eta^+ \le 1$.

To compare the DHFEs, Zhu *et al.* (2012a) introduced comparison laws as follows.

Definition 3 (Zhu *et al.* 2012a). Let $d_1 = \left(h_1, g_1\right)$ and $d_2 = \left(h_2, g_2\right)$ be any two DHFEs, $s(d_i) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma - \frac{1}{\# g} \sum_{\eta \in g} \eta \; (i = 1,2)$ the score function of $d_i (i = 1,2)$, and $p(d_i) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma + \frac{1}{\# g} \sum_{\eta \in g} \eta \; (i = 1,2)$ the accuracy function of $d_i (i = 1,2)$. The above mentioned $\#h$ and $\# g$ represented the quantity of components in h and g, respectively. Furthermore, Zhu *et al.* (2012a) defined the following rules.

If the inequality $s\left(d_1\right) < s\left(d_2\right)$ holds, then d_1 is inferior to d_2**,** denoted as $d_1 \prec d_2$.

If the equality $s\left(d_1\right) = s\left(d_2\right)$ holds, then:

i) d_1 is equivalent to d_2, denoted as $d_1 \sim d_2$, if $h\left(d_1\right) = h\left(d_2\right)$, and

ii) d_1 is superior to d_2, denoted as $d_1 \succ d_2$, if $h\left(d_1\right) > h\left(d_2\right)$.

Definition 4 (Zhu *et al.* 2012a). Suppose there is an objective set and marked by X, and let d, d_1 and d_2 be three any given DHFEs. Then:

$$d_1 \oplus d_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left\{ \gamma_1 + \gamma_2 - \gamma_1 \gamma_2 \right\}, \left\{ \eta_1 \eta_2 \right\} \right\};$$

$$d_1 \otimes d_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left\{ \gamma_1 \gamma_2 \right\}, \left\{ \eta_1 + \eta_2 - \eta_1 \eta_2 \right\} \right\};$$

$$nd = \bigcup_{\gamma \in h, \eta \in g} \left\{ \left\{ 1 - (1-\gamma)^n \right\}, \left\{ \eta^n \right\} \right\}, \quad n > 0;$$

$$d^n = \bigcup_{\gamma \in h, \eta \in g} \left\{ \left\{ \gamma^n \right\}, \left\{ 1 - (1-\eta)^n \right\} \right\}, \quad n > 0.$$

2. Aggregation operators for DHFEs

The weighted average (WA) and the weighted geometric (WG) operators are common aggregation operators used in information aggregation (Merigó 2012). They can be usefully employed in practical problems such as area of statistics, socioeconomic, and engineering world. Since their introduction, the WA and WG operators have been studied in a wide range of applications (Beliakov *et al.* 2007; Merigó, Casanovas 2011a, 2011b, 2011c, 2011d; Yager 1988, 2002, 2003, 2006, 2007, 2009a, 2009b; Zhao *et al.* 2010; Xu, Yager 2006; Wei 2009).

In this section, we have applied the WA and WG operators to dual hesitant fuzzy environment and introduced some aggregation operators to aggregate dual hesitant fuzzy information. To start with, we define the DHFWA operator and then propose the DHFWG operator. Based on the Definition 4, the DHFWA operator is defined as follows:

Definition 5. Let $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$ be a collection of DHFEs. A DHFWA operator is a mapping $D^n \to D$ such that:

$$\mathrm{DHFWA}\left(d_1,d_2,...,d_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\omega_j d_j\right)=\omega_1 d_1\oplus\omega_2 d_2\oplus\cdots\oplus\omega_n d_n, \tag{4}$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the measure of importance of d_j and ω are standardized. In particular, if $\omega=(\frac{1}{n},\frac{1}{n},...,\frac{1}{n})^T$, then the DHFWA operator degenerate into DHFA operator:

$$\mathrm{DHFA}\left(d_1,d_2,...,d_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\frac{1}{n}d_j\right)=\frac{1}{n}d_1\oplus\frac{1}{n}d_2\oplus\cdots\oplus\frac{1}{n}d_n. \tag{5}$$

Theorem 1. Suppose there is family of DHFEs $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$, then:

$$\mathrm{DHFWA}\left(d_1,d_2,...,d_n\right)=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\eta_j^{\ \omega_j}\right\}\right\}, \tag{6}$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the measure of importance of d_j and ω are standardized.

Proof: We first prove that Eq. (6) holds for n = 2.

$$\omega_1 d_1=\cup_{\gamma_1\in h_1,\eta_1\in g_1}\left\{\left\{1-(1-\gamma_1)^n\right\},\left\{\eta_1^{\ n}\right\}\right\}; \tag{7}$$

$$\omega_2 d_2=\cup_{\gamma_2\in h_2,\eta_2\in g_2}\left\{\left\{1-(1-\gamma_2)^n\right\},\left\{\eta_2^{\ n}\right\}\right\}. \tag{8}$$

Then,

$\mathrm{DHFWA}\left(d_1,d_2\right)=\omega_1 d_1\oplus\omega_2 d_2=$

$\cup_{\gamma_1\in h_1,\eta_1\in g_1,\gamma_2\in h_2,\eta_2\in g_2}\left\{\left\{2-(1-\gamma_1)^{\omega_1}-(1-\gamma_2)^{\omega_2}-(1-(1-\gamma_1)^{\omega_1})(1-(1-\gamma_2)^{\omega_2})\right\},\left\{\eta_1^{\omega_1}\eta_2^{\omega_2}\right\}\right\}=$

$$\cup_{\gamma_1\in h_1,\eta_1\in g_1,\gamma_2\in h_2,\eta_2\in g_2}\left\{\left\{1-\prod_{j=1}^{2}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{2}\eta_j^{\ \omega_j}\right\}\right\}. \tag{9}$$

If Eq. (6) is true when n = k, meaning:

$$\mathrm{DHFWA}\left(d_1,d_2,...,d_k\right)=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{k}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{k}\eta_j^{\ \omega_j}\right\}\right\}, \tag{10}$$

then, when n increase single unit, we can get:

$$\mathrm{DHFWA}\left(d_1,d_2,\ldots,d_{k+1}\right)=$$

$$\omega_1 d_1 \oplus \omega_2 d_2 \oplus \cdots \oplus \omega_n d_n \oplus \omega_{n+1} d_{n+1} = \left(\omega_1 d_1 \oplus \omega_2 d_2 \oplus \cdots \oplus \omega_n d_n\right) \oplus \omega_{n+1} d_{n+1} =$$

$$\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\omega_j} \right\} \right\} \oplus \cup_{\gamma_{k+1} \in h_{k+1}, \eta_{k+1} \in g_{k+1}} \left\{ \left\{ 1-(1-\gamma_{k+1})^{\omega_{k+1}} \right\}, \left\{ \eta_{k+1}{}^{\omega_{k+1}} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1-\prod_{j=1}^{k}\left(1-\gamma_j\right)^{\omega_j} + (1-(1-\gamma_{k+1})^{\omega_{k+1}}) - (1-\prod_{j=1}^{k}\left(1-\gamma_j\right)^{\omega_j})(1-(1-\gamma_{k+1})^{\omega_{k+1}}) \right\}, \left\{ \prod_{j=1}^{k+1} \eta_j^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1-\prod_{j=1}^{k+1}\left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{k+1} \eta_j^{\omega_j} \right\} \right\}. \quad (11)$$

In other words, Eq. (6) establishes when $n = k + 1$. Therefore, Eq. (6) establishes for any given n, completing the proof of Theorem 1.

Now, let us look at all sorts of excellent properties of the DHFWA operator.

Theorem 2. Suppose $d=\left(h,g\right)=\cup_{\gamma\in h,\eta\in g}\left\{\{\gamma\},\{\eta\}\right\}$ and $d_j=\left(h_j,g_j\right)(j=1,2,\ldots,n)$ be a collection of DHFEs. If for all j, $\gamma_j = \gamma$, $\eta_j = \eta$, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, γ is the element of HFS h, and η is the element of HFS g, then:

$$\mathrm{DHFWA}\left(d_1,d_2,\ldots,d_n\right) = d. \quad (12)$$

Proof: By Theorem 1, we have:

$$\mathrm{DHFWA}\left(d_1,d_2,\ldots,d_n\right) = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma \in h, \eta \in g} \left\{ \left\{ 1-\prod_{j=1}^{n}\left(1-\gamma\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma \in h, \eta \in g} \left\{ \left\{ 1-\left(1-\gamma\right)^{\sum_{j=1}^{n}\omega_j} \right\}, \left\{ \eta^{\sum_{j=1}^{n}\omega_j} \right\} \right\} = \cup_{\gamma \in h, \eta \in g} \left\{ \{\gamma\}, \{\eta\} \right\} = d, \quad (13)$$

completing the proof of Theorem 2.

Theorem 3. Suppose $d_j=\left(h_j,g_j\right)(j=1,2,\ldots,n)$ be a collection of DHFEs. If $d=\left(h,g\right)=\cup_{\gamma\in h,\eta\in g}\left\{\{\gamma\},\{\eta\}\right\}$ is a DHFE, γ_j are elements of HFS h_j, and η_j are elements of HFS g_j, then:

$$\mathrm{DHFWA}\left(d_1 \oplus d, d_2 \oplus d, \ldots, d_n \oplus d\right) = \mathrm{DHFWA}\left(d_1,d_2,\ldots,d_n\right) \oplus d. \quad (14)$$

Proof: Since for any j

$$d_j \oplus d = \cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ \gamma_j + \gamma - \gamma_j \gamma \right\}, \left\{ \eta_j \eta \right\} \right\} = \cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1-(1-\gamma_j)(1-\gamma) \right\}, \left\{ \eta_j \eta \right\} \right\}, \quad (15)$$

according to Theorem 1, we have:

$$\mathrm{DHFWA}(d_1 \oplus d, d_2 \oplus d, ..., d_n \oplus d) =$$

$$\cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left((1-\gamma_j)(1-\gamma) \right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} (\eta_j \eta)^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1 - (1-\gamma)^{\sum_{j=1}^{n}(\omega_j)} \prod_{j=1}^{n} (1-\gamma_j)^{\omega_j} \right\}, \left\{ \eta^{\sum_{j=1}^{n}(\omega_j)} \prod_{j=1}^{n} (\eta_j)^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1 - (1-\gamma) \prod_{j=1}^{n} (1-\gamma_j)^{\omega_j} \right\}, \left\{ \eta \prod_{j=1}^{n} (\eta_j)^{\omega_j} \right\} \right\}. \quad (16)$$

According to the operational laws of Definition 4, we can get:

$$\mathrm{DHFWA}\left(d_1, d_2, ..., d_n\right) \oplus d =$$

$$\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\omega_j} \right\} \right\} \oplus \cup_{\gamma \in h, \eta \in g} \left\{ \{\gamma\}, \{\eta\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1 - (1-\gamma)(1-(1-\prod_{j=1}^{n} \left(1-\gamma_j\right)^{\omega_j})) \right\}, \left\{ \eta \prod_{j=1}^{n} (\eta_j)^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \gamma \in h, \eta_j \in g_j, \eta \in g} \left\{ \left\{ 1 - (1-\gamma) \prod_{j=1}^{n} \left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \eta \prod_{j=1}^{n} (\eta_j)^{\omega_j} \right\} \right\}. \quad (17)$$

Thus,

$$\mathrm{DHFWA}(d_1 \oplus d, d_2 \oplus d, ..., d_n \oplus d) = \mathrm{DHFWA}(d_1, d_2, ..., d_n) \oplus d, \quad (18)$$

completing the proof of Theorem 3.

Theorem 4. Suppose $d_j = \left(h_j, g_j\right)(j = 1, 2, ..., n)$ be a family of DHFEs. If $r > 0$, then:

$$\mathrm{DHFWA}(rd_1, rd_2, \ldots, rd_n) = r\, \mathrm{DHFWA}(d_1, d_2, \ldots, d_n). \quad (19)$$

Proof: According to Definition 4, we have:

$$rd_j = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - (1-\gamma_j)^r \right\}, \left\{ \eta_j^{\,r} \right\} \right\}. \quad (20)$$

According to Theorem 1, we have:

$$\mathrm{DHFWA}(rd_1, rd_2, \ldots, rd_n) = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left(\left(1-\gamma_j\right)^r \right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} (\eta_j^{\,r})^{\omega_j} \right\} \right\} =$$

$$\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left(1-\gamma_j\right)^{r\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\,r\omega_j} \right\} \right\}; \quad (21)$$

$$r\, \mathrm{DHFWA}(d_1, d_2, \ldots, d_n) = r \left(\cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left(1-\gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\omega_j} \right\} \right\} \right) =$$

$$\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-(1-\left(1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right))^r\right\},\left\{\left(\prod_{j=1}^{n}\eta_j^{\omega_j}\right)^r\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-(\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j})^r\right\},\left\{\left(\prod_{j=1}^{n}\eta_j^{\omega_j}\right)^r\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{r\omega_j}\right\},\left\{\prod_{j=1}^{n}\eta_j^{r\omega_j}\right\}\right\}. \tag{22}$$

Thus:

$$\text{DHFWA}(rd_1,rd_2,\ldots,rd_n)=r\,\text{DHFWA}(d_1,d_2,\ldots,d_n). \tag{23}$$

According to Theorems 3 and 4, we can get Theorem 5 easily.

Theorem 5. Suppose $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$ be a family of DHFEs. If $r > 0$ and $d=\left(h,g\right)=\cup_{\gamma\in h,\eta\in g}\left\{\{\gamma\},\{\eta\}\right\}$ is a DHFE, then:

$$\text{DHFWA}(rd_1\oplus d,rd_2\oplus d,\ldots,rd_n\oplus d)=r\,\text{DHFWA}(d_1,d_2,\ldots,d_n)\oplus d. \tag{24}$$

Theorem 6. Suppose $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$ and $l_j=\left(m_j,n_j\right)(j=1,2,...,n)$ be two families of DHFEs, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, θ_j is the element of HFS m_j, and σ_j is the element of HFS n_j, then:

$$\text{DHFWA}(d_1\oplus l_1,d_2\oplus l_2,\ldots,d_n\oplus l_n)=\text{DHFWA}(d_1,d_2,\ldots,d_n)\oplus\text{DHFWA}(l_1,l_2,\ldots,l_n). \tag{25}$$

Proof: According to the operational laws of Definition 4, we have:

$$d_j\oplus l_j=\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{\gamma_j+\theta_j-\gamma_j\theta_j\right\},\left\{\eta_j\sigma_j\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{1-\left(1-\gamma_j\right)\left(1-\theta_j\right)\right\},\left\{\eta_j\sigma_j\right\}\right\}. \tag{26}$$

According to Theorem 1, we have:

$$\text{DHFWA}(d_1\oplus l_1,d_2\oplus l_2,\ldots,d_n\oplus l_n)=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(\left(1-\gamma_j\right)\left(1-\theta_j\right)\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\left(\eta_j\sigma_j\right)^{\omega_j}\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\prod_{j=1}^{n}\left(1-\theta_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\left(\eta_j\sigma_j\right)^{\omega_j}\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\xi_j\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right\},\left\{\prod_{j=1}^{n}\left(\eta_j\right)^{\omega_j}\prod_{j=1}^{n}\left(\sigma_j\right)^{\omega_j}\right\}\right\}; \tag{27}$$

$$\text{DHFWA}(d_1,d_2,\ldots,d_n)\oplus\text{DHFWA}(l_1,l_2,\ldots,l_n)=$$

$$\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\eta_j^{\ \omega_j}\right\}\right\}\oplus\cup_{\theta_j\in m_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\theta_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\sigma_j^{\ \omega_j}\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}$$

$$\left\{\left\{\left(1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right)+\left(1-\prod_{j=1}^{n}\left(1-\theta_j\right)^{\omega_j}\right)-\left(1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right)\left(1-\prod_{j=1}^{n}\left(1-\theta_j\right)^{\omega_j}\right)\right\},\left\{\prod_{j=1}^{n}\left(\eta_j\right)^{\omega_j}\prod_{j=1}^{n}\left(\sigma_j\right)^{\omega_j}\right\}\right\}=$$

$$\cup_{\gamma_j\in h_j,\theta_j\in m_2,\eta_j\in g_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\xi_j\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right\},\left\{\prod_{j=1}^{n}\left(\eta_j\right)^{\omega_j}\prod_{j=1}^{n}\left(\sigma_j\right)^{\omega_j}\right\}\right\}. \quad (28)$$

Thus,

$$\text{DHFWA}(d_1\oplus l_1,d_2\oplus l_2,\ldots,d_n\oplus l_n)=\text{DHFWA}(d_1,d_2,\ldots,d_n)\oplus\text{DHFWA}(l_1,l_2,\ldots,l_n), \quad (29)$$

completing the proof of Theorem 6.

Theorem 7. Suppose $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$ and $l_j=\left(m_j,n_j\right)(j=1,2,...,n)$ be two families of DHFEs, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, θ_j are elements of HFS m_j, and σ_j are elements of HFS n_j. If for all j, $\gamma_j\ge\theta_j$ and $\eta_j\le\sigma_j$, then,

$$\text{DHFWA}(d_1,d_2,...,d_n)\ge\text{DHFWA}(l_1,l_2,...,l_n). \quad (30)$$

Proof: Since,

$$\text{DHFWA}\left(d_1,d_2,...,d_n\right)=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\eta_j^{\ \omega_j}\right\}\right\}; \quad (31)$$

$$\text{DHFWA}(l_1,l_2,...,l_n)=\cup_{\theta_j\in m_j,\sigma_j\in n_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\theta_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\sigma_j^{\ \omega_j}\right\}\right\}. \quad (32)$$

Furthermore, since $\gamma_j\ge\theta_j$ and $\eta_j\le\sigma_j$ for all j, then according to the definition of the comparison laws of DHFS, we know that Theorem 7 is true.

Aggregated geometric mean (Saaty 1980; Willet, Sharda 1991; Benjamin *et al.* 1992; Yu 2012; Yu *et al.* 2012) and DHFWA operator, we define here a DHFWG operator.

Definition 6. Suppose $d_j=\left(h_j,g_j\right)(j=1,2,...,n)$ be a family of DHFEs. A DHFWG operator is:

$$\text{DHFWG}\left(d_1,d_2,...,d_n\right)=d_1^{\ \omega_1}\oplus d_2^{\ \omega_2}\oplus\cdots\oplus d_n^{\ \omega_n}, \quad (33)$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the measure of importance of d_j and ω are standardized. In particular, if $\omega=(\frac{1}{n},\frac{1}{n},...,\frac{1}{n})^T$, then the DHFWG operator degenerate into DHFG operator:

$$\text{DHFG}\left(d_1,d_2,...,d_n\right)=d_1^{\ \frac{1}{n}}\oplus d_2^{\ \frac{1}{n}}\oplus\cdots\oplus d_n^{\ \frac{1}{n}}. \quad (34)$$

Theorem 8. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a family of DHFEs. Then,

$$\text{DHFWG}\left(d_1, d_2, ..., d_n\right) = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ \prod_{j=1}^{n} \gamma_j^{\omega_j} \right\}, \left\{ 1 - \prod_{j=1}^{n} (1-\eta_j)^{\omega_j} \right\} \right\}, \quad (35)$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the measure of importance of d_j and ω are standardized.

Theorem 9. Let $d = \left(h, g\right) = \cup_{\gamma \in h, \eta \in g} \left\{\{\gamma\}, \{\eta\}\right\}$ and $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a collection of DHFEs. If for all j, $\gamma_j = \gamma$, $\eta_j = \eta$,, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, γ is the element of HFS h, η is the element of HFS g, then:

$$\text{DHFWG}(d_1, d_2, ..., d_n) = d. \quad (36)$$

Theorem 10. Suppose $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a family of any given DHFEs. If $d = \left(h, g\right) = \cup_{\gamma \in h, \eta \in g} \left\{\{\gamma\}, \{\eta\}\right\}$ is a DHFE, γ_j are elements of HFS h_j, η_j are elements of HFS g_j, then:

$$\text{DHFWG}(d_1 \otimes d, d_2 \otimes d, ..., d_n \otimes d) = \text{DHFWG}(d_1, d_2, ..., d_n) \otimes d. \quad (37)$$

Proof: The proof of Theorem 10 is similar to that of Theorem 3.

Theorem 11. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a collection of DHFEs. If $r > 0$, then:

$$\text{DHFWG}(rd_1, rd_2, \ldots, rd_n) = (\text{DHFWG}(d_1, d_2, \ldots, d_n))^r. \quad (38)$$

Proof: The proof of Theorem 11 is similar to that of Theorem 4.

Using Theorems 10 and 11, we can get Theorem 12 easily.

Theorem 12. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a collection of DHFEs. If $r > 0$ and $d = \left(h, g\right) = \cup_{\gamma \in h, \eta \in g} \left\{\{\gamma\}, \{\eta\}\right\}$ is a DHFE, then:

$$\text{DHFWG}\left(\left(d_1\right)^r \otimes d, \left(d_2\right)^r \otimes d, \ldots, \left(d_n\right)^r \otimes d\right) = (\text{DHFWG}(d_1, d_2, \ldots, d_n))^r \otimes d. \quad (39)$$

Theorem 13. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ and $l_j = \left(m_j, n_j\right)(j=1,2,...,n)$ be two collections of DHFEs, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, θ_j is the element of HFS m_j, σ_j is the element of HFS n_j, then:

$$\text{DHFWG}(d_1 \otimes l_1, d_2 \otimes l_2, \ldots, d_n \otimes l_n) = \text{DHFWG}(d_1, d_2, \ldots, d_n) \otimes \text{DHFWG}(l_1, l_2, \ldots, l_n). \quad (40)$$

Theorem 14. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ and $l_j = \left(m_j, n_j\right)(j=1,2,...,n)$ be two collections of DHFEs, where γ_j are elements of HFS h_j, η_j are elements of HFS g_j, θ_j is the element of HFS m_j, and σ_j is the element of HFS n_j. If for all j, $\gamma_j \geq \theta_j$ and $\eta_j \leq \sigma_j$, then,

$$\text{DHFWG}(d_1, d_2, ..., d_n) \geq \text{DHFWG}(l_1, l_2, ..., l_n). \quad (41)$$

In order to understand the relationship between the DHFWA and DHFWG operators, we introduce the following Theorem.

Theorem 15. Let $d_j = \left(h_j, g_j\right)(j=1,2,...,n)$ be a collection of DHFEs. Then,

$$\text{DHFWG}\left(d_1, d_2, \cdots, d_n\right) \leq \text{DHFWA}\left(d_1, d_2, \cdots, d_n\right). \quad (42)$$

3. Selection of human resources outsourcing suppliers

Consider a multi-criteria decision-making problem under uncertainty (Hu *et al.* 2013; Rolland 2013; Wang *et al.* 2013; Ertay *et al.* 2013). Let $Y=\{Y_1,Y_2,...,Y_m\}$ be the bunch of alternative schemes and $C=\{C_1,C_2,...,C_n\}$ be the family of criteria. Assuming that the experts provide the assessment information under the criterion C_j for the alternative Y_i using a DHFEs γ_{ij}, based on which, the matrix $D=(\gamma_{ij})_{m\times n}$ can be constructed. Next, based on DHFWA and DHFWG operators, we give a decision-making procedure using DHFSs as follows:

Step 1. Aggregate the DHFEs γ_{ij} for each alternative Y_i using the DHFWA (or DHFWG) operator.

$$\text{DHFWA}\left(d_1,d_2,...,d_n\right)=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right\},\left\{\prod_{j=1}^{n}\eta_j^{\omega_j}\right\}\right\} \tag{43}$$

or

$$\text{DHFWG}\left(d_1,d_2,...,d_n\right)=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\left\{\left\{\prod_{j=1}^{n}\gamma_j^{\omega_j}\right\},\left\{1-\prod_{j=1}^{n}(1-\eta_j)^{\omega_j}\right\}\right\}. \tag{44}$$

Step 2. Sort the alternative schemes by Definition 3.

$$S(d_i)=\frac{1}{\#h}\sum\nolimits_{\gamma\in h}\gamma-\frac{1}{\#g}\sum\nolimits_{\eta\in g}\eta,\ i=1,2,\cdots,m. \tag{45}$$

Then, the bigger the value of $S(\gamma_i)$, the larger the overall DHFE γ_i will be, so choose the alternative Y_i ($i=1,2,\cdots,m$).

It is quite common for enterprises to outsource human resource services from a third-party provider while concentrating on their core businesses. A company defines requirements for human resources, and the human resources outsourcing firm will attempt to provide associated services to meet such requirements. Some human resources outsourcing firms are generalists, providing all sorts of services, while others may be more specialized, focusing on specific areas such as payrolls and recruitments. Therefore, an enterprise can outsource all human resources tasks or only some of them depending on its business need and how much control it wish to retain over its human resources functions. Typical services provided by human resources outsourcing firms include organizational structure planning and personnel requirements, recruitment, training and development, and so on. Let us consider a foreign company ABC that recently started its core business in an industrial park. As a new comer in the region, ABC consider it is better to outsource HR services from an outsider provider. ABC views HR outsourcing as a strategic tool for getting the right people for its core business. ABC is now facing a decision which HR service provider should be selected to take its HR responsibilities.

After full consideration, they choose three evaluation criteria: enterprise size and background (C_1), outsources service quantity (C_2), and service quality (C_3). The criterion weight vector is supposed as $w=(0.3,0.4,0.3)^T$. The evaluation information on the alternatives $x_i(i=1,2,...,4)$ under the criterion $C=\left\{C_1,C_2,C_3\right\}$ is represented by the DHFEs $d_{ij}=\cup_{\gamma_{ij}\in h_{ij},\eta_{ij}\in g_{ij}}\left\{\left\{\gamma_{ij}\right\},\left\{\eta_{ij}\right\}\right\}$, $0\le\gamma_{ij},\eta_{ij}\le 1$ and $0\le\gamma_{ij}^{+}+\eta_{ij}^{+}\le 1$. The dual hesitant fuzzy decision information given by experts is shown in Table 1.

Table 1. Evaluation information

	C_1	C_2	C_3
x_1	{{0.5,0.6,0.7},{0.2,0.3}}	{{0.6,0.7},{0.1,0.2}}	{{0.7, 0.8},{0.3}}
x_2	{{0.5,0.6},{0.1,0.2}}	{{0.5,0.6},{0.1,0.2,0.3,0.4}}	{{0.7,0.8,0.9},{0.1}}
x_3	{{0.5,0.6},{0.3,0.4}}	{{0.4,0.5},{0.2,0.3,0.4,0.5}}	{{0.7 },{0.2,0.3}}
x_4	{{0.7,0.8},{0.1,0.2}}	{{0.5,0.6,0.7},{0.2,0.3}}	{{0.6,0.8},{0.1,0.2}}

If we use the DHFWA operator, the main steps are as follows:

Step 1. Utilize the DHFWA operator (Eq. (6)) to fuse all the DHFEs d_{ij} in the ith line of D and obtain the synthesized DHFEs d_i.

d_1 ={{0.6904, 0.7485, 0.7367, 0.7862, 0.7104, 0.7648, 0.7538, 0.8000, 0.7344, 0.7842, 0.7741, 0.8165}, {0.1866, 0.2462, 0.2297, 0.3031}}

d_2 ={{0.6134, 0.7070, 0.6860, 0.7620}, {0.2259, 0.2980, 0.3259, 0.3505, 0.2366, 0.3121, 0.3413, 0.3671}}

d_3 = {{0.7178, 0.7862, 0.7361, 0.8000}, {0.2980, 0.3669, 0.3259, 0.4012, 0.3728, 0.4590, 0.3933, 0.4842, 0.3249, 0.4000, 0.3552, 0.4373, 0.4064, 0.5004, 0.4287, 0.5278}}

d_4 = {{0.5988, 0.6741, 0.6331, 0.7020, 0.6730, 0.7344, 0.7115, 0.7656, 0.7361, 0.7856, 0.7648, 0.8089}, { 0.1320, 0.1625, 0.1741, 0.2144, 0.1835, 0.2259, 0.2421, 0.2980}}

Step 2. Calculate the scores of d_i $(i=1,2,3,4)$, respectively, as

$s(d_1)=0.5169$, $s(d_2)=0.3849$, $s(d_3)=0.3549$, $s(d_4)=0.5117$.

Since:

$s(d_1)>s(d_4)>s(d_2)>s(d_3)$,

we have

$x_1 \succ x_4 \succ x_2 \succ x_3$

The best option is candidate x_1.

If we use the DHFWG operator, the main steps are as follows:

Step 1′. Utilize the DHFWG operator (Eq. (37)) to fuse all the DHFEs $d_{ij}(i=1,2,3,4)$ in the ith line of D and obtain the synthesized DHFEs d_i'.

d_1' = {{0.6587, 0.6823, 0.6948, 0.7198, 0.6957, 0.7207, 0.7339, 0.7602, 0.7286, 0.7548, 0.7686, 0.7962}, {0.2307, 0.2661, 0.2944, 0.3268}}

d_2' = {{0.3933, 0.4122, 0.4287, 0.4494}, {0.3268, 0.4000, 0.4422, 0.4898, 0.3825, 0.4496, 0.4883, 0.5320}}

d_3' = {{0.6415, 0.7198, 0.6776, 0.7602}, {0.3150, 0.3716, 0.3631, 0.4158, 0.4808, 0.5238, 0.5586, 0.5951, 0.3459, 0.4000, 0.3919, 0.4422, 0.5043, 0.5453, 0.5785, 0.6134}}

d_4' = {{0.5842, 0.6369, 0.6284, 0.6850, 0.6684, 0.7286, 0.6300, 0.6867, 0.6776, 0.7387, 0.7207, 0.7857}, {0.1414, 0.1712, 0.2347, 0.2613, 0.2038, 0.2314, 0.2903, 0.3150}}

Step 2′. Calculate the scores of d_i' $(i=1,2,3,4)$, respectively, as:

$s(d_1')=0.4467$, $s(d_2')=-0.018$, $s(d_3')=0.2345$, $s(d_4')=0.4498$.

Since

$$s(d_4) > s(d_1) > s(d_3) > s(d_2),$$

we have

$$x_4 \succ x_1 \succ x_3 \succ x_2 .$$

The best option is candidate x_4.

The optimal decision has changed, the sort result obtained using the DHFWG operator is different from that obtained using the DHFWA operator. The DHFWA operator focuses on the impact of the overall data while the DHFWG operator highlights the role of individual data.

Concluding remarks

As a generalization of FSs, DHFSs give us an additional possibility for depicting imperfect knowledge. In this paper, we have developed a DHFWA operator and a DHFWG operator for information aggregation that extends two of the broadly applicable aggregation operators (the WA and WG operators) to accommodate situations, in which the input information is DHFEs. We also studied various properties of the proposed operators and have illustrated their application to the selection of human resources outsourcing suppliers in a dual hesitant fuzzy environment.

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