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Fuzzy sets and fuzzy logic in multi-criteria decision making. The 50th anniversary of prof. Lotfi Zadeh's theory

DYNAMIC FUZZY MULTIPLE CRITERIA DECISION MAKING FOR PERFORMANCE EVALUATION

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Abstract. The paper proposes a dynamic fuzzy multiple criteria decision making (DFMCDM) method. The method considers the integrated weight of the decision makers with the subjective and objective preference and the effect of time weight. In the proposed method, a mathematical programming model is used to determine the integrated weight, and a basic unit-interval monotonic (BUM) function based approach is used to calculate the time weight. In addition, a distance measure of membership function is introduced to effectively measure the degree of difference between the alternatives in the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS). Finally, a numerical example is introduced to illustrate the proposed method.

Keywords: dynamic fuzzy multiple criteria decision making, integrated weight, basic unit-interval monotonic function, time weight, fuzzy TOPSIS.

JEL Classification: C44, C61, D81.

Introduction

Multiple criteria decision making (MCDM) is widely used to deal with the ranking and selection of alternatives with respect to multiple attributes. In most of the economical, financial, political and industrial decision problems, the selection and evaluation of solutions are usually regarded as the typical MCDM problems (Hashemkhani Zolfani *et al.* 2013; Lu *et al.* 2013; Kou *et al.* 2014a, 2014b, 2014c; Ondemir, Gupta 2014).

However, some decision problems usually involve imprecise, uncertain, indefinite and subjective data, which could cause the decision process to become more complex and challengeable. Moreover, each decision maker has limited information, different prefer-

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ence structures and complex decision making background while some attributes are more abstract. In reality, to reflect the attribute information, most decision makers are usually willing to give the linguistic variables or fuzzy variables rather than crisp values. In order to solve these problems, fuzzy set theory was proposed by Zadeh (Zadeh 1965, 1978), and established a theoretical foundation for decision makers. One of applications of the fuzzy theory is in fuzzy multiple criteria decision making (FMCDM). The fuzzy set theory is an efficient way for FMCDM methods to model uncertainty and imprecision in terms of linguistic variables. For example, Bellman and Zadeh (1970) proposed the basic decision-making processes in a fuzzy environment. Chu and Lin (2009) gave an extension to fuzzy MCDM where linguistic values were represented by fuzzy number and the Riemann integral based mean of removals was suggested to rank all the final fuzzy evaluation values for final decision making. Razavi Hajiagha *et al.* (2013) proposed a complex proportional assessment method for MCDM in an interval-valued intuitionistic fuzzy environment. Zavadskas *et al.* (2014) proposed an extended version of WASPAS method which could be applied in uncertain decision making environment. Among the proposed methods, the accuracy of weighted sum and weighted product models were also improved. Some other FMCDM were also proposed by the scholars (Buckley 1985; Chang 1996; Fenton, Wang 2006; Mahdavi *et al.* 2008; Torfi *et al.* 2010; Dursun, Karsak 2013; Kannan *et al.* 2014). However, there are some shortcomings in these methods. For example, some authors only considered the subjective weight or objective weight when the weight was calculated, and some authors did not fully consider the inherent fuzzy character of fuzzy numbers and the information contained in membership function when calculating the distances of the fuzzy numbers. Moreover, in many real life situations, such as multi-period investment and personnel dynamic examination, the decision information are generally provided by decision makers at the different periods. Therefore, it is necessary to develop the dynamic fuzzy multiple criteria decision making (DFMCDM) to deal with the dynamic performance evaluation problems.

In the process of FMCDM, the weights of attributes or experts play a significant role because they directly affect the accuracy of the decision-making and the ranking results of the alternatives. The evaluation of criteria usually entails diverse opinions and meanings, thus it cannot be assumed that each evaluation criterion is of equal importance (Chen *et al.* 2003). Generally, the methods of determining the weights of attributes can be roughly grouped into two categories: subjective methods and objective methods. The subjective methods determine the weights of attributes in terms of the subjective preference or judgment of the decision makers, including the direct rating method (Bottomley, Doyle 2001; Roberts, Goodwin 2002), Delphi method (Hwang, Yoon 1981), Analytic Hierarchy Process (AHP) (Saaty 1977, 1980; Kou, Lin 2014) and others (Deng *et al.* 2004; Figueira, Roy 2002). These methods are also applied in decision making process (Peng *et al.* 2011; Ergu, Kou 2012; Kou *et al.* 2012; Ergu *et al.* 2013; Siozinyte *et al.* 2014). The objective methods determine the weights of attributes by using objective decision matrix information or solving mathematical models, including entropy method (Shannon, Weaver 1947), Data Envelopment Analysis (DEA) (Charnes *et al.* 1978), multiple programming (Srinivasan, Shocker 1973), ideal point method (Hwang, Yoon 1981) and their extended methods (Ma *et al.* 1999; Xu

2004; Wei 2008; Dejus, Antucheviciene 2013; Yano 2014; Zhu, Xu 2014). However, both subjective methods and objective methods have their advantages and disadvantages. For example, subjective methods can take full advantage of subjective opinions of experts, but they are difficult to eliminate preconception caused by lack of knowledge or experience of the decision makers; objective methods have strong mathematical and theoretical basis, and the evaluation results do not depend on human factors, but they do not reflect the subjective preferences of decision makers, and ignore the accumulation of knowledge and experience of experts. In order to make accurate and scientific decisions, the decision makers are usually required to give qualitative or quantitative assessments for determining the performance and relative importance of the evaluation criteria. Therefore, some integrated weight methods have been proposed by many references (Wang, Lee 2009; Nabavi-kerizi *et al.* 2010; Zhang, Zhou 2011; Parameshwaran *et al.* 2015). In this paper, a new integrated weight method is proposed based on subjective and objective information. In the method, the subjective weight is given by expects and the objective weight is calculated by a modified entropy weighting method. Then, an optimization model is used to determine the integrated weight.

In many real life situations, such as personnel dynamic selection and multi-period investment, the decision processes are generally provided by decision makers at different periods. Thus, it is necessary to develop the dynamic evaluation method to deal with the performance evaluation. Recently, the research on dynamic evaluation problems has received many attentions. For example, Chen and Li (2011) proposed a dynamic multi-attribute decision making model based on the triangular intuitionistic fuzzy numbers. Lin *et al.* (2008) developed a dynamic multi-attribute decision making model that takes the TOPSIS technique as the main structure, integrating the concepts of grey number and Minkowski distance function into it to deal with the uncertain information and aggregate the multi-period evaluations. Park *et al.* (2013) proposed an extension of the VIKOR method for dynamic intuitionistic fuzzy multiple attribute decision making. Xu (2008) proposed a multi-period multi-attribute decision making (MP-MADM) problems where the decision information are provided by decision maker(s) at different periods. Xu and Yager (2008) proposed a dynamic multi-attribute decision making problems where all the decision information about attribute values takes the form of the intuitionistic fuzzy numbers collected at different periods. In this paper, a dynamic fuzzy multiple criteria decision method (DFMCDM) is proposed based on TOPSIS. In DFMCDM, the time weight is unknown, and a basic unit-interval monotonic (BUM) function based approach (Yager 1996, 2004) is used to calculate the time weight. Besides, a distance measure of membership function is applied to effectively measure the degree of difference between the alternatives in the DFMCDM. Since the distance measurement of membership function contains the inherent fuzzy character of fuzzy numbers and the information contained in membership function, it makes the results of the assessment more accurate. Finally, a numerical example is used to illustrate the proposed method and the results show that the proposed method is effective for performance evaluation.

1. Definitions and theorems

In the section, some basic definitions and theorems of fuzzy sets and fuzzy numbers are reviewed from Kaufmann and Gupta (1988).

Definition 1 (Kaufmann, Gupta 1988)**.** A positive triangular fuzzy number $\tilde{A}$ can be defined as $\tilde{A} = (a,b,c), 0 \le a \le b \le c$, if the membership function $\mu_{\tilde{A}}: R \to [0,1]$ is defined as follows:

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{x-a}{b-a}, & a < x < b \\ 1, & x = b \\ \dfrac{x-c}{b-c}, & b < x < c \\ 0, & \text{others} \end{cases} . \quad (1)$$

Now, given any two triangular fuzzy numbers $\tilde{A} = (a_1,a_2,a_3)$, $\tilde{B} = (b_1,b_2,b_3)$ and a positive real number λ, some main operations of the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be expressed as follows:

1) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,
2) $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$,
3) $\lambda\tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3)$,
4) $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$.

Because a fuzzy number cannot be simply described by the completely independent of the n-dimensional coordinates $(a_1, a_2, \cdots, a_n)$, it should be portrayed by a specific membership function. Mahdavi *et al.* (2008) proposed a distance measure of membership function, and the distance contains the inherent fuzzy character of fuzzy numbers and the information contained in membership function (He *et al.* 2010). The specific form, called distance measure of membership function $D_{2,1/2}$, is given as follows (Mahdavi *et al.* 2008):

$$d(\tilde{A},\tilde{B}) = \sqrt{\frac{(a_1-b_1)^2 + 2(a_2-b_2)^2 + (a_3-b_3)^2 + (a_1-b_1)(a_2-b_2) + (a_2-b_2)(a_3-b_3)}{6}} . \quad (2)$$

2. Integrated weight

For convenience, the alternatives are expressed as $x = \{x_1, x_2, \cdots, x_m\}$ and the evaluation attributes are expressed as $c = \{c_1, c_2, \cdots, c_n\}$. It is assumed that the attributes are additively independent. $\tilde{x}_{ij}$ is the assessed value of the attribute c_j of the alternative x_i and expressed by a triangular fuzzy number in this paper. The different values of $\tilde{x}_{ij}$ can be represented by means of a decision making matrix $\tilde{V} = (\tilde{x}_{ij})_{m \times n}$. In order to eliminate the difference of the attribute index on the dimension, each attribute index is normalized by:

$$\tilde{x}'_{ij} = \begin{cases} \tilde{x}_{ij} \Big/ \sum_{i=1}^{m} \tilde{x}_{ij}, \forall i \in M, j \in I_1, \\ (1/\tilde{x}_{ij}) \Big/ \sum_{i=1}^{m} (1/\tilde{x}_{ij}), \forall i \in M, j \in I_2, \end{cases} ,(3)$$

where I_2 is associated with a set of benefit criteria, I_2 is associated with a set of cost criteria and $M = \{1,2,\cdots,m\}$.

Let $\mu = (\mu_1, \mu_2, \cdots, \mu_n)^T$ be the weight of the evaluation attributes which is given by subjective judgement of the expert, and $\omega=(\omega_1, \omega_2 \cdots \omega_n)^T$ be the objective weight of the evaluation attributes. In order to determine the objective weight, a modified entropy weighting method is proposed.

Entropy concept initially proposed by Shannon (Shannon, Weaver 1947) is a measure of uncertainty in information formulated in terms of probability theory. In MCDM methods, Shannon's entropy concept is used in weighting calculation method. Entropy weight is a parameter to describe the disorder degree of a system. It can measure the amount of the useful information with the provided data. The greater the entropy value, the smaller the entropy weight, then the smaller the different alternatives in the specific attribute, and the less information the specific attribute gives, and the less important the attribute becomes in decision making process. The basic operation of entropy value given by Shannon is shown as follows:

$$E_j = -k\sum_{i=1}^{m} p_{ij} \ln p_{ij} \text{ , } i = 1,2,\cdots m \text{ , } j = 1,2,\cdots,n, \tag{4}$$

where k is a constant and usually taken $k = 1/\ln m$, and $p_{ij} \ln p_{ij}$ is also equal to zero when p_{ij} is equal to zero.

Since entropy is a measure of uncertainty in information formulated in terms of probability theory, it shows that the degree of uncertainty is greater if the data is more dispersed. We should adjust the computing process of the entropy by introducing the deviation degree, and then give a modified entropy weighting method to determine the objective weights. Let the normalized decision matrix be $\tilde{V}' = [\tilde{x}'_{ij}]_{m\times n}$ and $D(\tilde{x}'_{ij}, \tilde{x}'_{kj})$ be the deviation degree between $\tilde{x}'_{ij}$ and $\tilde{x}'_{kj}$ in the normalized decision matrix $\tilde{V}'$, where $D(\tilde{x}'_{ij}, \tilde{x}'_{kj})$ is given by $D_{2,1/2}$. For attribute $\tilde{x}'_j$, the deviation degree between alternative x_i and any other alternative could be calculated by:

$$D_{ij} = \sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj}), i \in M, j \in N, \tag{5}$$

the deviation degree between all alternatives and any other alternative can be calculated by

$$D_j = \sum_{i=1}^{m} D_{ij} = \sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj}), i \in M. \tag{6}$$

The processes of the modified entropy weighting method are shown as follows

Step 1. Calculate the entropy value of a modified operation:

$$E_j = -k\sum_{i=1}^{m} \frac{D_{ij}}{D_j} \ln \frac{D_{ij}}{D_j} \text{ , } i = 1,2,\cdots m \text{ , } j = 1,2,\cdots,n, \tag{7}$$

where k is a constant, it is usually taken $k = 1/\ln m$.

Step 2. Calculate the degree of differences d_j:

$$d_j = 1 - E_j \text{, } j = 1,2,\cdots,n. \tag{8}$$

Step 3. Calculate the objective weight vector ω_j:

$$\omega_j = \frac{d_j}{\sum_{j=1}^{n} d_j}, \ j = 1,2,\cdots,n. \tag{9}$$

According to the characteristics of the subjective weight and objective weight, a integrated weight is calculated by an optimization model.

Let the integrated weight vector be $W = (W_1, W_2, \cdots, W_n)^T$, and $W = \alpha\mu + \beta\omega$, where $\alpha \geq 0$ and $\beta \geq 0$ are the coefficients of linear representation of the integrated weight vector, respectively, and both satisfy the unitized constraint conditions $\alpha^2 + \beta^2 = 1$. In order to calculate the integrated weight, the values of α and β should be determined. Thus, a calculation method according to maximum deviations principle is introduced in this paper. The basic idea of this method is that if the attribute values of all alternatives under the attribute $\tilde{x}'_{ij}$ have larger difference, which shows the attribute plays an important role in ranking the alternatives. In other words, the greater deviation degree of the attribute should be given greater sort weight. Therefore, the choice of the attribute weight vector should make all attributes maximize overall deviation degree of all alternatives. The calculating process is as follows.

Let the normalized decision matrix be $\tilde{V}' = [\tilde{x}'_{ij}]_{m\times n}$. For the attribute $\tilde{x}'_j$, the deviation degree between alternative x_i and any other alternative can be calculated by:

$$D_{ij} = \sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj}), \ i \in M, j \in N, \tag{10}$$

the deviation degree between all alternatives and any other alternative is calculated by

$$D_j = \sum_{i=1}^{m} D_{ij} = \sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj}), \ i \in M. \tag{11}$$

Since the choice of the integrated weight vector W should make all attributes maximize overall deviation degree of all alternatives, a linear programming model based on maximum deviations is constructed as follows:

$$\max D(W) = \max \sum_{j=1}^{n} D_j W_j = \max \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj}) W_j = \max \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj})(\alpha\mu_j + \beta\omega_j),$$

s.t. $\alpha^2 + \beta^2 = 1$, $\alpha \geq 0$, $\beta \geq 0$,

where $D(\tilde{x}'_{ij}, \tilde{x}'_{kj})$ represents the deviation degree, and $D(\tilde{x}'_{ij}, \tilde{x}'_{kj})$ is given by $D_{2, 1/2}$.

In order to solve the model, the Lagrange function is constructed and shown below,

$$L(\alpha, \beta, \lambda) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj})(\alpha\mu_j + \beta\omega_j) + \frac{1}{2}\lambda(\alpha^2 + \beta^2 - 1),$$

where λ is Lagrange multiplier.

Let $\frac{\partial L(\alpha,\beta,\lambda)}{\partial \alpha} = 0$, $\frac{\partial L(\alpha,\beta,\lambda)}{\partial \lambda} = 0$, $\frac{\partial L(\alpha,\beta,\lambda)}{\partial \beta} = 0$, we have

$$\begin{cases} \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}_{ij}^{'},\tilde{x}_{kj}^{'})\mu_j + \lambda\alpha = 0 \\ \alpha^2+\beta^2-1=0 \\ \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}_{ij}^{'},\tilde{x}_{kj}^{'})\omega_j + \lambda\beta = 0 \end{cases}.$$

Let $D_1 = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}_{ij}^{'},\tilde{x}_{kj}^{'})\mu_j$, $D_2 = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}_{ij}^{'},\tilde{x}_{kj}^{'})\omega_j$, since $\alpha \geq 0$ and $\beta \geq 0$, solving the model, we have:

$$\begin{cases} \alpha = \dfrac{D_1}{\sqrt{D_1^2 + D_2^2}} \\ \lambda = -\sqrt{D_1^2 + D_2^2}. \\ \beta = \dfrac{D_2}{\sqrt{D_1^2 + D_2^2}} \end{cases} \tag{12}$$

Therefore, the integrated weight can be calculated by:

$$W_j = \frac{D_1}{\sqrt{D_1^2 + D_2^2}}\mu_j + \frac{D_2}{\sqrt{D_1^2 + D_2^2}}\omega_j. \tag{13}$$

Normalizing the weight vector, we have:

$$W_j^* = \frac{W_j}{\sum_{j=1}^{n} W_j}. \tag{14}$$

3. Time weight

In the DFMCDM, the decision making process is influenced by the change of time. So it is important to determine the weight of time. A basic unit-interval monotonic (BUM) function based approach (Yager 1996, 2004) is given to determine the weight of time.

Definition 2 (Yager 1996)**.** The function $Q:[0,1]\to[0,1]$ is a BUM function, where 1) $Q(0) = 0$; 2) $Q(1) = 1$; 3) $Q(x) \geq Q(y)$, if $x > y$.

Based on the BUM function, the time weight can be determine as follows:

$$\lambda(t_k) = Q\left(\frac{k}{p}\right) - Q\left(\frac{k-1}{p}\right),\ k = 1,2,\cdots,p. \tag{15}$$

In order to calculate the time weight $\lambda(t_k)$, based on Xu (2009), we suppose that $Q(x) = \dfrac{e^{\alpha x}-1}{e^{\alpha}-1}$, $\alpha > 0$, then the time weight can be given as follows:

$$\lambda(t_k) = \frac{e^{\frac{\alpha k}{p}}(1-e^{-\frac{\alpha}{p}})}{e^{\alpha}-1},\ k = 1,2,\cdots,p. \tag{16}$$

4. DFMCDM

In the decision making process, the fuzzy information and multi-period evaluation are of common occurrence. Therefore, a DFMCDM is proposed in the paper. Usually, a fuzzy number should contain some information and be portrayed by a specific membership function. Since the distance $D_{2,\frac{1}{2}}$ contains the inherent fuzzy character of fuzzy numbers and the information contained in membership function, it can be introduced to calculate the distance between attributes. The proposed DFMCDM procedure based on TOPSIS is given as follows.

Step 1. Construct the fuzzy decision matrix and normalize the fuzzy decision matrix. The normalized fuzzy decision matrix is denoted by $\tilde{V}' = [\tilde{x}'_{ij}]_{m\times n}$, where $\tilde{x}'_{ij} = (\tilde{x}'^{l}_{ij}, \tilde{x}'^{m}_{ij}, \tilde{x}'^{r}_{ij})$ is a triangular fuzzy number.

Step 2. Calculate the integrated weight of the attribute by:

$$W_j = \frac{D_1}{\sqrt{D_1^2 + D_2^2}}\mu_j + \frac{D_2}{\sqrt{D_1^2 + D_2^2}}\omega_j,$$

where $D_1 = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj})\mu_j$, $D_2 = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} D(\tilde{x}'_{ij}, \tilde{x}'_{kj})\omega_j$, μ is the subjective weight of the evaluation attributes and ω is the objective weight of the evaluation attributes.

Step 3. Calculate the time weight by:

$$\omega(t_k) = \frac{e^{\frac{\alpha k}{p}}(1 - e^{-\frac{\alpha}{p}})}{e^{\alpha} - 1}.$$

Step 4. Construct dynamic weighted fuzzy normalized decision matrix $\tilde{Z}' = [\tilde{f}'_{ij}]_{m\times n}$, where

$$\tilde{f}'_{ij} = \sum_{k=1}^{p} W_{jk}\cdot \tilde{x}'_{ijk}\cdot \omega(t_k). \tag{17}$$

Step 5. Select the fuzzy positive ideal solution $\tilde{f}^{+}$ and fuzzy negative ideal solution $\tilde{f}^{-}$, where

$$\tilde{f}_j^{+} = (\tilde{f}_j^{+l}, \tilde{f}_j^{+m}, \tilde{f}_j^{+r}) = (\max_i \tilde{f}_{ij}^{l}, \max_i \tilde{f}_{ij}^{m}, \max_i \tilde{f}_{ij}^{r}); \tag{18}$$

$$\tilde{f}_j^{-} = (\tilde{f}_j^{-l}, \tilde{f}_j^{-m}, \tilde{f}_j^{-r}) = (\min_i \tilde{f}_{ij}^{l}, \min_i \tilde{f}_{ij}^{m}, \min_i \tilde{f}_{ij}^{r}). \tag{19}$$

Step 6. Calculate the distance of each alternative from the positive ideal solution and negative ideal solution:

$$\tilde{S}_i^{+} = \sum_{j=1}^{n} D_{2,\frac{1}{2}}(\tilde{f}_{ij}, \tilde{f}_j^{+}),\ i = 1,2,\cdots,m; \tag{20}$$

$$\tilde{S}_i^{-} = \sum_{j=1}^{n} D_{2,\frac{1}{2}}(\tilde{f}_{ij}, \tilde{f}_j^{-}),\ i = 1,2,\cdots,m. \tag{21}$$

Step 7. Calculate the similarities to ideal solution:

$$\tilde{C}_i = \frac{\tilde{S}_i^{-}}{\tilde{S}_i^{+} + \tilde{S}_i^{-}},\ i = 1,2,\cdots,m. \tag{22}$$

Step 8. Rank the preference order according to $\tilde{C}_i$ in descending order.

5. Numerical example

In this section, a numerical example adapted from Xu (2009) is used to illustrate the proposed method. An investment bank needs to invest a sum of money in the best option from four possible enterprises, denoted as x_1, x_2, x_3 and x_4, respectively. Three attributes c_1: social benefits; c_2: economic benefits; c_3: environment pollution are taken into consideration, where c_1 and c_2 are the benefit criteria, and c_3 is the cost criteria, the value of the evaluated attribute is given by the triangular fuzzy number. The performance of the enterprises are evaluated in three years, denoted as t_1, t_2 and t_3. Assume that the subjective weights of attributes at the year t_k denote as $\mu(t_1) = (0.45, 0.35, 0.20)^T$, $\mu(t_2) = (0.45, 0.30, 0.25)^T$, and $\mu(t_3) = (0.40, 0.30, 0.30)^T$, respectively. In order to select the best enterprise, the DFMCDM is applied as follows.

Step 1. Construct the fuzzy decision matrix $x(t_k)$ and normalize the fuzzy decision matrix $\tilde{x}(t_k)$ in every year in Tables 1–6.

Table 1. Fuzzy decision matrix $x(t_1)$

	x_1	x_2	x_3	x_4
c_1	(0.6,0.7,0.8)	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.5,0.6,0.7)
c_2	(0.6,0.7,0.8)	(0.5,0.6,0.7)	(0.7,0.8,0.9)	(0.6,0.7,0.8)
c_3	(0.3,0.4,0.5)	(0.2,0.3,0.4)	(0.4,0.5,0.6)	(0.3,0.4,0.5)

Table 2. Fuzzy decision matrix $x(t_2)$

	x_1	x_2	x_3	x_4
c_1	(0.6,0.7,0.8)	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.5,0.6,0.7)
c_2	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.6,0.7,0.8)	(0.5,0.6,0.7)
c_3	(0.2,0.3,0.4)	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.2,0.3,0.4)

Table 3. Fuzzy decision matrix $x(t_3)$

	x_1	x_2	x_3	x_4
c_1	(0.5,0.6,0.7)	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.5,0.6,0.7)
c_2	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.7,0.8,0.9)	(0.5,0.6,0.7)
c_3	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.2,0.3,0.4)	(0.2,0.3,0.4)

Table 4. Normalized fuzzy decision matrix $\tilde{x}(t_1)$

	x_1	x_2	x_3	x_4
c_1	(0.187,0.250,0.333)	(0.219,0.286,0.375)	(0.187,0.250,0.333)	(0.156,0.214,0.292)
c_2	(0.187,0.250,0.333)	(0.156,0.214,0.292)	(0.219,0.286,0.375)	(0.187,0.250,0.333)
c_3	(0.141,0.242,0.408)	(0.176,0.323,0.612)	(0.118,0.194,0.306)	(0.141,0.242,0.408)

Table 5. Normalized fuzzy decision matrix $\tilde{x}(t_2)$

	x_1	x_2	x_3	x_4
c_1	(0.182,0.241,0.320)	(0.212,0.276,0.360)	(0.212,0.276,0.360)	(0.152,0.207,0.280)
c_2	(0.219,0.286,0.375)	(0.187,0.250,0.333)	(0.187,0.250,0.333)	(0.156,0.214,0.292)
c_3	(0.158,0.299,0.577)	(0.105,0.179,0.288)	(0.126,0.224,0.385)	(0.158,0.299,0.577)

Table 6. Normalized fuzzy decision matrix $\tilde{x}(t_3)$

	x_1	x_2	x_3	x_4
c_1	(0.161,0.222,0.304)	(0.226,0.296,0.391)	(0.194,0.259,0.348)	(0.161,0.222,0.304)
c_2	(0.212,0.276,0.360)	(0.182,0.241,0.320)	(0.212,0.276,0.360)	(0.152,0.207,0.280)
c_3	(0.105,0.179,0.288)	(0.126,0.224,0.385)	(0.158,0.299,0.577)	(0.158,0.299,0.577)

Step 2. Calculate the objective weight and integrated weight of the attribute in every year, the objective weights in every year are as follows:

$$\omega(t_1) = (0.319,\ 0.319,\ 0.362)^T;$$
$$\omega(t_2) = (0.285,\ 0.581,\ 0.134)^T;$$
$$\omega(t_3) = (0.252,\ 0.509,\ 0.239)^T.$$

The integrated weights in every year are as follows:

$$W(t_1) = (0.379,0.333,0.288)^T;$$
$$W(t_2) = (0.374,0.429,0.197)^T;$$
$$W(t_3) = (0.329,0.401,0.270)^T.$$

Step 3. Calculate the time weight by formula (16) and suppose that $\alpha = 0.5$, we have:

$$\lambda(t_1) = 0.28, \lambda(t_2) = 0.33, \lambda(t_3) = 0.39\ .$$

Step 4. Construct dynamic weighted fuzzy normalized decision matrix $\tilde{Z}' = [\tilde{f}_{ij}']_{m\times n}$ in Table 7.

Table 7. Dynamic weighted fuzzy normalized decision matrix $\tilde{Z}' = [\tilde{f}_{ij}']_{m\times n}$

	x_1	x_2	x_3	x_4
c_1	(0.063,0.085,0.114)	(0.078,0.102,0.134)	(0.071,0.094,0.129)	(0.056,0.077,0.105)
c_2	(0.082,0.107,0.140)	(0.069,0.093,0.124)	(0.080,0.105,0.138)	(0.063,0.086,0.116)
c_3	(0.033,0.058,0.101)	(0.034,0.061,0.109)	(0.034,0.062,0.110)	(0.038,0.070,0.131)

Step 5. Select the fuzzy positive ideal solution and fuzzy negative ideal solution by formula (18) and formula (19), we obtain:

$$\tilde{f}^{+} = [(0.078,0.102,0.134),(0.082,0.107,0.140),(0.038,0.070,0.131)]^T\ ;$$
$$\tilde{f}^{-} = [(0.056,0.077,0.105),(0.063,0.086,0.116),(0.033,0.058,0.101)]^T\ .$$

Step 6. Calculate the distance of each alternative from the positive ideal solution and negative ideal solution by formula (20) and formula (21), we have:

$$\tilde{S}_i^+ = (0.0349, 0.0265, 0.0210, 0.0472), \ \tilde{S}_i^- = (0.0293, 0.0376, 0.0434, 0.0170).$$

Step 7. Calculate the similarities to ideal solution by formula (22), we get:

$$\tilde{C}_i = (0.4567, 0.5866, 0.6743, 0.2647).$$

Step 8. Rank the preference order based on the values of the similarities, we obtain:

$$x_3 \succ x_2 \succ x_1 \succ x_4,$$

thus, the best enterprise is x_3.

The selection alternative of the proposed method is consistent with Xu (2009), which shows the feasibility and practicality of the proposed method. Furthermore, since both subjective weight and objective weight are considered, the proposed method not only avoids the human factor deviation, but also takes full advantage of the subjective opinions of experts. At the same time, the time weight is considered in the proposed method, it is convenient to select the right alternative for the decision-makers.

Conclusions

In many real decision problems, it is difficult for decision makers to choose their preferences in the form of uncertain information because a result of vague knowledge about the preference of alternatives. Accordingly, it is necessary to study the decision making problems under fuzzy environment in modern decision analysis. In order to improve the fairness and reliability of decision-making and as possible as meet the people's decision-making characteristics, a DFMCDM method is proposed in this paper. Because the subjective weight, objective weight and time weight are considered, the proposed method not only avoids the deviation of the human factors, but also takes full account of the preferences of decision makers and the influence of time factor. In calculating the distance of fuzzy numbers, a distance measure of membership function is applied to effectively measure the degree of difference between the alternatives, which improves the decision-making effects and makes the decision-making process more reasonable.

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