FUZZY TWO-PERSON GAMES FOR TECHNICAL DECISIONS

F. Peldschus

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1. The problem

Technical problems of industry, agriculture and other areas are taking on dimensions that do no longer allow satisfactory solutions to be arrived at by the use of previously employed tools. These are complex and interwoven problems involving requirements made by different groups of persons. Attempts to interpret such problems as conflict situations and make them accessible to game-theoretic solutions have already been reported in the literature (for example, Refs. 1 and 2). In those cases, use was made of well-known solution procedures assuming definite gain functions. However, it should be noted that it is often quite difficult to obtain precise information for practical applications. In many cases, it is only possible to give rough values. However, since precise information is required to be assumed, this will have an influence upon the quality of the solution obtained. In the following discussion, the interconnection of the theory of matrix games and the theory of fuzzy sets offers the possibility of taking account of what is known as fuzziness.

2. Classical matrix games

In their classical form, matrix games are described by two nonempty sets S_1 and S_2, the strategy sets of players I and II, and a profit function A (S_1, S_2) defined for the Cartesian product $S_1 \times S_2$. Use is generally made of the symbolic notation

$$\Gamma = (S_1, S_2, A). \tag{1}$$

For a solution, the two players orientate themselves by the payoff bounds, namely:

$$\underline{a}(S_1) = \inf a(s_1, s_2) \quad \text{warranty bound for } s_1 \in S_1$$
$$s_2 \in S_2$$

and

$$\bar{a}(S_2) = \sup a(s_1, s_2) \quad \text{warranty bound for } s_2 \in S_2$$
$$s_1 \in S_1$$

Choosing the optimum warranty bounds, we obtain:

$$a_*(S_1, S_2) = \max \inf a(s_1, s_2)$$
$$s_1 \in S_1 \quad s_2 \in S_2$$

and

$$a^*(S_1, S_2) = \min \sup a(s_1, s_2).$$
$$s_2 \in S_2 \quad s_1 \in S_1$$

For a solution, it is necessary to orientate to a balance situation that manifests itself as a saddle point.

For the game $\Gamma = (S_1, S_2, A)$ a saddle point will be obtained, if and only if the expressions

$$\max \inf a(s_1, s_2) \quad \text{and} \quad \min \sup a(s_1, s_2)$$
$$s_1 \in S_1 \quad s_2 \in S_2 \qquad s_2 \in S_2 \quad s_1 \in S_1$$

exist and are equal, i.e., if

$$\max \inf a(s_1, s_2) = \min \sup a(s_1, s_2).$$
$$s_1 \in S_1 \quad s_2 \in S_2 \qquad s_2 \in S_2 \quad s_1 \in S_1$$

Since the strategy sets are finite, the expressions do always exist. (Accordingly, it is possible for inf and sup to be replaced by min and max, respectively).

The balance strategies of player 1 are those strategies $s_1 \in S_1$, for which the s_2 infimum reaches the maximum relative to s_1. Analogously, the balance strategies for player 2 are those strategies $s_2 \in S_2$, for which the s_1 supremum reaches the minimum relative to s_2.

The criterion

$$\max \min a(s_1, s_2) = \min \min a(s_1, s_2) = v \tag{2}$$
$$s_1 \in S_1 \quad s_2 \in S_2 \qquad s_2 \in S_2 \quad s_1 \in S_1$$

is the well-known min-max principle (Ref. 3), with the value v being the game value.

3. Fuzzy sets

The theory of fuzzy sets, which is based upon investigations reported by Zadeh (Ref. 4), involves a mathematical description of vague (inexact, fuzzy) elements, with the vagueness of information resulting not from the stochastic character of the systems, but from the lack of uniqueness or selectivity thereof. Accordingly, the answer to the question whether an element is associated with a fuzzy set or not will not be in the form of a YES-OR-NO decision but requires a carefully gradated judgment of its association. The degree of association of defined elements is determined by an association function that must correspond to certain mathematical definitions, axioms, and operational rules.

A fuzzy set A in X is a set of ordered pairs.

$$A = \{x, \ _{/}u_A(x)/ \ x \in X\} \qquad (3)$$

In the above expression, $_{/}u_A(x)$ is the degree of association of x with the fuzzy set A.

$_{/}u_A : X \to R$ means association function and is a real-valued function.

Usually, the range of values of $_{/}u_A$ is restricted to the closed interval (Ref. 1).

For a fuzzy decision, the association function $_{/}u_A(x)$ indicates the degree to which each element x will satisfy the respective requirements.

An element $x \in A$ signifies an optimum fuzzy decision if x possesses the maximum degree of association with A.

A widely held view ascribable to Bellmann and Zadeh is that a fuzzy decision is defined as an average of the fuzzy sets for fuzzy objectives Z and the fuzzy restrictions R.

For the cut set of two fuzzy sets Z and R the association function is defined, pointwise, by the operator

$$_{/}u_A(x) = Min\left[\ _{/}u_Z(x), \ _{/}u_R(x)\right]. \qquad (4)$$

4. Association function

Constituting the association function is the totality of values $_{/}u_A(x)$ for all of the elements x from X involved.

Various concepts can be resorted to for a determination of the association function. Frequently, piecewise linear association functions are considered.

In the interval $x_m > x_o$ we obtain:

$$_{/}u_A(x) = \begin{cases} 0 & \text{for } x \leq x_o \\ 1 - \dfrac{x - x_o}{x_m - x_o} & \text{for } x_o \leq x \leq x_m \\ 1 & \text{for } x_m \leq x \end{cases} \qquad (5)$$

In addition to the linear slope of an association function, consideration is also given to an S-shaped behaviour. The advantage of nonlinear association functions lies in the fact that the transition to the "nonassociated" and "completely associated" ranges takes place far more harmoniously. The interpolating cubic spline function proved itself extremely useful for practical examples within the framework of multicriterion decision problems (Ref. 5). Using the supporting points $(x_o, 0)$, $(x_D, 0.5)$, $(x_M, 1)$ and the boundary conditions $_{/}u'_A(x_o) = \ _{/}u'_A(x_M) = 0$ we obtain two thirddegree polynomials which are joined together in x_D in a twice continuously differentiable form. This gives the following set-up for $x_o < x_M$:

$$_{/}u_A(x) = \begin{cases} 0 & \text{for } x \leq x_o \\ Ax^3 + Bx^2 + Cx + D & \text{for } x_o \leq x \leq x_D \\ Ex^3 + Fx^2 + Gx + H & \text{for } x_D \leq x \leq x_M \\ 1 & \text{for } x_M \leq x \end{cases} \qquad (6)$$

The coefficients $(A, ..., H)$ are calculated from a system of equations (7) derived from the requirements of $_{/}u_A(x)$ such as continuity, existence of the first and second derivatives, and choise of supporting points and boundary conditions.

$$(G_1) \quad Ax_o^3 + Bx_o^2 + Cx_o + D = 0$$

$$(G_2) \quad Ax_D^3 + Bx_D^2 + Cx_D + D = 0.5$$

$$(G_3) \quad Ex_D^3 + Fx_D^2 + Cx_D + H = 0.5$$

$$(G_4) \quad Ex_M^3 + Fx_M^2 + Cx_M + H = 1$$

$$(G_5) \quad 3Ax_o^2 + 2Bx_o + C = 0$$

$$(G_6) \quad 3Ex_M^2 + 2Fx_M + G = 0$$

$$(G_7) \quad 3Ax_D^2 + 2Bx_D + C - 3Ex_D^2 - 2Fx_D - G = 0$$

$$(G_8) \quad 6Ax_D + 2B - 6Ex_D - 2F = 0$$

$$(7)$$

This system of equations $\{(G_1), ..., (G_8)\}$ is nonsingular and has a unique solution.

For $_{/}u_A(x)$ to be $C[0,1]$ and monotonic in x_j it is still necessary for the condition

$$-1 + \sqrt{2} \le \frac{|x_M - x_D|}{|x_D - x_o|} \le 1 + \sqrt{2} \qquad (8)$$

to be satisfied, which is usually the case. Problems will be encountered only if x_D is in the region of x_o or x_M, respectively.

5. Fuzzy matrix games

The games described in Section 2 of this paper will now be considered in connection with the theory of fuzzy sets. Approaches have already been reported in, for example, Refs. 6, 7, 8, and 9. However, lack of operationalization did not yet allow them to be made practical use of. The classical theory of games assumes that interpersonal conflict situations can be precisely described mathematically. The assumption made in this context is that the elements of a particular game can be represented as sharply defined sets. This involves orientation to an analysis of given mathematical structures. However, if more exact requirements are made of modeling, then it is no longer possible for the existence of sharp sets to be postulated.

The elements of the game are affected by various sources of fuzziness. The gain or payoff function is not always defined numerically or sharply, respectively. It is formulated semantically and, at the same time, fuzzily, such as excellent, good, or sufficient reliability, durability, or resistance. The strategies employed by players are, in general, marked by different levels of significance and intensity, respectively. These and other conditions are the reason for the need to include the theory of fuzzy sets in the solution concept of the theory of games.

For two players employing defined strategy sets that are wholly or partially comprised of fuzzy information the fuzzy game $\Gamma_f u$ can be written as follows:

$$\Gamma_f u = \left\{ (S_{1i}, {}_f u_{1i}); \ (S_{2i}, {}_f u_{ij}); \ (a_{ij}, {}_f \tilde{u}_{ij}) \right\} \qquad (9)$$

With S_{1i}: for $i = 1, \ldots, m$ Strategies of player I
${}_f u_{1i}$: for $i = 1, \ldots, m$ Association function for the strategies of player I
S_{2i}: for $j = 1, \ldots, n$ Strategies of player II
${}_f u_{ij}$: for $i = 1, \ldots, m$ Association function for
$j = 1, \ldots, n$ the strategies of player II relative to the strategies of player I

a_{ij}: for $i = 1, \ldots, m$ Payoff or gain function
$j = 1, \ldots, n$
${}_f u_{ij}$: for $i = 1, \ldots, m$ Association function for
$j = 1, \ldots, n$ the payoff function

The transition from game Γ to game $\Gamma_f u$ is in three steps. A detailed description of these steps is given in Ref. 10.

First step:

A fuzzy set is defined for the set of strategies of player I. The set of strategies and criteria quantitatively describing the strategies are assumed to be known. An association function (6) is calculated for each of the criteria, i.e., standard values are relativized to give values of association. Thus, we obtain, for each strategy of player I, a value of association for the different criteria. The collection of values of association is in the form of an arithmetic mean (Laplace criterion).

$$_f u_{1i} = \frac{1}{L} \sum_{l=1}^{L} {}_f u_{il} \qquad (10)$$

The values ${}_f u_{1i}$ are calculated in the matrix (11).

	K_1	K_2	...	K_l	...	K_L	${}_f u_{1i}$
S_{11}	${}_f u_{11}$	${}_f u_{12}$	...	${}_f u_{1l}$	...	${}_f u_{1L}$	${}_f u_{11}$
S_{12}	${}_f u_{21}$	${}_f u_{22}$	...	${}_f u_{2l}$	...	${}_f u_{2L}$	${}_f u_{12}$
$\vdots$	$\vdots$			$\vdots$		$\vdots$	$\vdots$
S_{1i}	${}_f u_{i1}$	${}_f u_{i2}$	...	${}_f u_{il}$	...	${}_f u_{iL}$	${}_f u_{1i}$
$\vdots$	$\vdots$			$\vdots$		$\vdots$	$\vdots$
S_{1m}	${}_f u_{m1}$	${}_f u_{m2}$	...	${}_f u_{ml}$	...	${}_f u_{mL}$	${}_f u_{1m}$

(11)

Second step:

Step no. 2 is concerned with the strategies for player II.

Fuzzy sets are defined for the set of strategies of player II, and the values of association are calculated according to (6). The mapping of sets is in the form of a matrix, initially signifying a basic matrix (12) for the game to be resolved. Whereas the matrix in step no.1 was used for an additive purpose, the basic matrix is to be interpreted gametheoretically.

EXAMPLE: INVESTIGATION OF DIFFERENT VARIANTS FOR A CABLE LINE FOR THE CHEMICAL INDUSTRY

Table 1: Internal and External Factors.

Parameters / Variant	Internal Factors									External Factors					
	Site Plan Parameters					Construction Parameters				Environmental Parameters			Execution Parameters		
	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
1. Cable Basin 2. Accessible Channel 3. Cable Trough 4. Cable Laying 5. Concrete Threshold 6. Cable Bridge 7. Cable Cover 8. Cable - Pipe - Bridge 9. Cable Fence	SPACE DEMANDS - CONSTRUCTION	CROSSING POSSIBILITIES	CONNECTION POSSIBILITIES	EXTENSION POSSIBILITIES	CABLE SALVAGE	SPACE DEMANDS - SITE	CAPACITY DEMANDS	CONSTRUCTION TIME	CONSTRUCTION COSTS	CORROSION RISK	FIRE RISK	EXPLOSION RISK	CONSTRUCTION ERRORS	BREAKDOWN DURATION	PROTECTION MEASURES

Table 2: Fuzzy Games.

SII / SI	S_{21}		S_{22}		S_{23}		S_{24}		S_{25}		S_{26}		α_i
$(S_{11}; 0{,}576)$	0,690	0,576	1,000	0,576	1,000	0,576	0,151	0,151	0,107	0,107	1,000	0,576	0,151
$(S_{12}; 0{,}498)$	0,642	0,498	0,777	0,498	0,970	0,498	0,970	0,498	0,890	0,498	0,000	0,000	0,000
$(S_{13}; 0{,}506)$	0,777	0,506	0,890	0,506	0,970	0,506	0,970	0,506	0,499	0,499	0,107	0,107	0,107
$(S_{14}; 0{,}708)$	0,890	0,708	1,000	0,708	1,000	0,708	0,107	0,107	0,151	0,151	1,000	0,708	0,107
$(S_{15}; 0{,}748)$	0,970	0,748	0,642	0,642	0,642	0,642	0,642	0,642	1,000	0,748	0,890	0,748	0,642
$(S_{16}; 0{,}729)$	0,107	0,107	0,151	0,151	0,107	0,107	1,000	0,729	0,970	0,729	0,642	0,642	0,107
$(S_{17}; 0{,}588)$	0,890	0,588	1,000	0,588	1,000	0,588	0,777	0,588	0,000	0,000	1,000	0,588	0,000
$(S_{18}; 0{,}549)$	0,107	0,107	0,000	0,000	0,000	0,000	0,970	0,549	0,970	0,549	0,499	0,499	0,000
$(S_{19}; 0{,}723)$	0,499	0,499	0,151	0,151	0,151	0,151	1,000	0,723	1,000	0,723	0,042	0,042	0,042

$$
\begin{array}{c|ccccc}
 & S_{21} & S_{22} & \cdots & S_{2j} & \cdots & S_{2n} \\
\hline
S_{11} & {}_{/}u_{11} & {}_{/}u_{12} & \cdots & {}_{/}u_{1j} & \cdots & {}_{/}u_{1n} \\
S_{12} & {}_{/}u_{21} & {}_{/}u_{2j} & \cdots & {}_{/}u_{2j} & \cdots & {}_{/}u_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\
S_{1i} & {}_{/}u_{i1} & {}_{/}u_{i2} & \cdots & {}_{/}u_{1j} & \cdots & {}_{/}u_{in} \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\
S_{1m} & {}_{/}u_{m1} & {}_{/}u_{m2} & \cdots & {}_{/}u_{mj} & \cdots & {}_{/}u_{mn}
\end{array}
\tag{12}
$$

Third step:

Step no.3 is a summary of steps nos. 1 and 2. This is an average of the strategy sets of players I and II, with Min being chosen as logic operator as mentioned above (4).

$$
{}_{/}\tilde{u}_{ij} = Min\left({}_{/}u_{1i},\ u_{ij} \right)
\tag{13}
$$

The fuzzy game matrix (14) proper is obtained as a result. Resolution is on the min-max principle (2) taken over from the classical theory of games.

$$
\begin{array}{c|cccccc}
 & S_{21} & S_{22} & \cdots & S_{2j} & \cdots & S_{2n} \\
\hline
S_{11} & /\tilde{u}_{11} & /\tilde{u}_{12} & \cdots & /\tilde{u}_{1j} & \cdots & /\tilde{u}_{1n} \\
S_{12} & /\tilde{u}_{21} & /\tilde{u}_{22} & \cdots & /\tilde{u}_{2j} & \cdots & /\tilde{u}_{2n} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \\
S_{1i} & /\tilde{u}_{i1} & /\tilde{u}_{i2} & \cdots & /\tilde{u}_{ij} & \cdots & /\tilde{u}_{in} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \\
S_{1m} & /\tilde{u}_{m1} & /\tilde{u}_{m2} & \cdots & /\tilde{u}_{mj} & \cdots & /\tilde{u}_{mn}
\end{array}
\qquad (14)
$$

6. Results and applications

The algorithm developed for fuzzy matrix games is a fuzzy concept for multi-criterion decisions. This concept was developed to enable both internal and external influential variables to be considered. Internal influential variables have an experiential character and will be effective until the system is made use of (building or manufacturing phase, respectively). External influential variables describe a new quality.

They have a predictive character and represent the phase of utilization. Thus, an algorithm is available which also enables quality features having a hierarchical structure to be aggregated, with different phases being allowed.

Practical investigations have already been discussed for studies of variants of cable routes in the chemical industry and for water supply systems (Ref. 10).

The strategies of player I include the constructional variants. These are studied with due consideration of the following aspects: Territorial and layout parameters such as space requirements, absence of crossings, possible connections and building parameters such as time and amount of building, possible extensions, capital costs. They represent what is known as internal influential variables. Interbalancing of parameters is allowed so that application of a compensatory operator (10) is justifiable. The result of step no. 1 of the fuzzy matrix game $\Gamma_j u$ is used to assess the strategies of player 1 by values of association. This serves to express the effective components of the strategies employed by player I. The strategies of player II include the use-related influences, i.e., resistance to failure, depreciations, and operating and power costs. They represent what is known as external influential variables.

Interaction of the strategies of player I with the strategies of player II is by the agency of the minimum operator (3). A matrix describing the fuzzy game $\Gamma_j u$ is obtained as a result.

The resolution of game $\Gamma_j u$ has a strategic character. It is used for the above-discussed examples (Ref. 10) of the selection of an optimum variant with due consideration of several criteria and the satisfaction of practical conditions that are beset by uncertainty, lack of information, and fuzziness.

Fuzzy matrix games provide numerous new possibilities of tackling practical engineering, economic, investment planning, and other problems. The resolution of fuzzy matrix games constitutes a new quality of decisions representing a high degree of complexity.

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NERAIŠKIEJI DVIEJŲ ASMENŲ ŽAIDIMAI DĖL TECHNINIŲ SPRENDIMŲ PRIĖMIMO

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S a n t r a u k a

Nagrinėjami neraiškieji dviejų asmenų žaidimai dėl techninių sprendimų priėmimo. Iš pradžių analizuojami klasikiniai matriciniai žaidimai, po to neraiškiosios aibės. Taip pat nagrinėjami neraiškieji matriciniai žaidimai. Atlikus šių metodų analizę, buvo pateiktos išvados. Pabrėžta, kad pasiūlytas metodas atveria naujas galimybes sprendžiant visą eilę praktinių inžinerinių, ekonominių, investicinių ir kitų uždavinių.

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