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OPTIMIZATION OF ELASTIC-PLASTIC GEOMETRICALLY NON-LINEAR LIGHTWEIGHT STRUCTURES UNDER STIFFNESS AND STABILITY CONSTRAINTS

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Abstract. An actual structural design, especially that of lightweight structures, must evaluate strength, stiffness and stability constraints. A designed structure must satisfy optimality criteria. One faces known difficulties when trying to implement several from above mentioned requirements into optimization problem for further successful numerical realisation. A method to formulate the optimization problem, incorporating all above described criterions, mathematical model and algorithm to solve it numerically, taking into account the geometrically non-linear structural behaviour are presented for truss type structure. In each optimization cycle the member forces obtained in the previous optimization cycle via elastic-plastic non-linear analysis procedure are employed to obtain the new optimal design values. During the optimization procedures, the tension members are assumed to be loaded up to the yield limit, compression members are assumed to be stressed up to their critical limits, the nodal displacements are restricted to limited magnitudes in prescribed directions. Design examples are presented to demonstrate the application of the algorithm.

Keywords: elastic-plastic structure, optimization, stiffness and stability constraints, geometrical non-linearity, tangent stiffness method, finite element discrete model.

1. Introduction

The main target of a engineer-designer is to create a structure, following the certain codified requirements, to be safe in respect of external actions and achieve certain economical effect. The lightweight structures are widely applied in current engineering practice, but those are sensitive to various kinds of deformations. The actual displacements of such structures must be evaluated applying the geometrical non-linear behaviour. Thus, the actual design of lightweight structures must be provided via the geometrical non-linear analysis in order to describe their actual response to external loading.

The structural design is a selection of certain structural members to satisfy optimality and safety criteria in respect of maintenance (strength, stiffness, stability) requirements. It is important to note that stiffness and stability constraints often dominate versus strength conditions in actual design of optimal structures [1]. When solving the limit equilibrium problem [2–4], the structural parts deformations or nodal displacements can exceed the fixed admissible magnitudes and/or fail due to stability loss. Thus, the main optimal structural design problem is to be stated as the structural optimization problem under the stiffness and stability requirements. A certain type discrete structure stiffness constraints are formulated in form of inequalities of nodal displacements versus limited displacement magnitudes; the stability constraints – in form of inequalities of critical forces versus compressive internal forces of structural units.

There optimality criterion-based method [5] employ the single (most critical constraint idea) in order to avoid the calculation of large Lagrange multipliers sets. Applying the above technique one transforms the constrained problem into an unconstrained one via the Lagrange multipliers. The necessary condition for the local constrained optimum is derived from the Lagrange function stationarity conditions [6, 7]. But no one can guarantee that the Kuhn-Tucker complementarity conditions are to be satisfied applying the method. The identified local extremum can fail then.

The above lack can be overcome when the more general method, based on the simultaneous application of the mathematical programming theory and that of extreme energy principles [1, 8, 9] is applied. The obtained optimization problem is the multiextremal one, as it constraints contain the complementarity condition. The last defines the field of solution to be discrete, ie consisting of the certain number of separate points. The investigation [10] proposed the complementarity conditions to include into objective function of the optimization problem [1, 11]. The latter in case of convex yield conditions result in convex admissible set of solutions and unique optimal solution. But analysing the problem from the point of mathematical programming, the practical solution of the problem is hard to obtain, even unavailable.

The task of the present investigation is the further development of the optimization problem solution methods, applying extreme complementary energy principle [1, 4] for elastic-plastic lightweight structures, subjected by external load, under presence strength, stiffness and stability constraints.

We assume the optimal structure to be in the state prior to plastic failure, resulting from stiffness and stability constraints. Some structural elements can be in a full plastic state, some partially plastically deformed or in elastic state. The optimization problem procedure realises iterative procedures, for each iteration applying in previous iteration identified set of limit forces, to satisfy requirements identified via previous analysis problem solution). The optimization procedure is continued until certain convergence.

2. Lightweight structure optimization problem mathematical model and algorithm

The structure optimization problem under the stress, stiffness and stability constraints consists of three principal parts of optimization cycle:

1. Determining truss-type structure elastic inner forces $\mathbf{S}_e$ and displacements $\mathbf{u}_e$ taking into account non-linear behaviour of external loading versus displacements (geometrical non-linearity) for presribed cross-sectional areas.
2. Defining the actual stress and strain state of the structure, applying the magnitudes of the limit forces $\mathbf{S}_0$ and critical forces $\mathbf{S}_{cr}$ due to the solution (extremal point) obtained in previous optimization cycle via analysis problem solution.
3. Optimizing areas of bars (conditioning the limit $\mathbf{S}_0$ and critical $\mathbf{S}_{cr}$ forces) to satisfy strength, stiffness and stability constraints.

Each part consists of separate problems to be solved one after another during each iteration until convergence. The iterative optimization procedure is conditioned by the circumstance that elastic forces and displacements from one side and limit and critical forces from the other side depend on the actual cross-sectional areas of bars; being as input and output data of optimization iteration parts 1 and 3.

2.1. Evaluation of elastic forces and displacements by the tangent stiffness method

If large displacements of structure appear when loaded, its is described as the non-linear one, ie strain - displacement relationships contain the non-linear terms.

To identify the elastic magnitudes of all internal forces, selected into the vector

$$\mathbf{S}_e = (S_{e,j})^T \equiv (S_{e,1}, S_{e,2}, ..., S_{e,n})^T ,$$

and that of joint displacement

$$\mathbf{u_e} = (u_{e,i})^T \equiv (u_{e,1}, u_{e,2}, ..., u_{e,m})^T ,$$

one must solve the following problem:

$$[K_t]\mathbf{u} = \mathbf{F} , \quad (1)$$

where $[K_t]$ = $[K_e]$ + $[K_g]$ is overall tangent stiffness matrix of structure; $[K_e]$ is linear elastic overall stiffness matrix; $[K_g]$ is geometric stiffness matrix; n – total number of truss-type structure bars and m – number of global displacements.

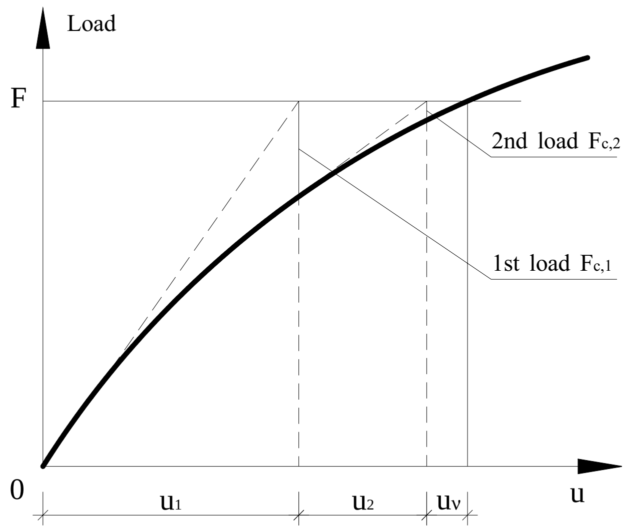

Fig 1. Computational procedure

The computation technique principle employed to determine the structure non-linear response values, graphical view is presented in Fig 1. The applied method realises the Newton-Raphson iterations, chosen in the way to satisfy truss-type structure equilibrium equations (1) obtained for nodes. Applying the Newton-Raphson method for certain load magnitude F the iterations υ are provided to eliminate the unbalanced (compensating) joint forces, resulting from unbalance of joint external forces and internal joint forces, ie:

$$\mathbf{F}_{c,\upsilon} = \mathbf{F} - \mathbf{F}_{s,\upsilon} . \quad (2)$$

The above tolerance appears, when the actual problem is linearised in the iteration under consideration, ie the tangent stiffness matrix is created due to the actual nodal displacements, identified during the previous iteration.

Consider the tangent stiffness method realisation steps from the starting one, ie $[K_g] = [0]$:

1. Define initial joint displacements $\mathbf{u}_0$ aplying the linear elastic analysis equations:

$$[K_e]\mathbf{u}_0 = \mathbf{F} . \quad (3)$$

Note that elastic stiffness matrix remains constant per optimization cycle. The matrix $[K_e]$ is obtained by the assemblage of the element elastic stiffness matrix (as suggested in [12])

$$[K_{e,j}] = \frac{E A_j}{l_j} \begin{bmatrix} [\bar{k}_{e,j}] & -[\bar{k}_{e,j}] \\ -[\bar{k}_{e,j}] & [\bar{k}_{e,j}] \end{bmatrix} \quad (4)$$

in the global coordinate system, where

$$[\bar{k}_{e,j}] = \begin{bmatrix} \cos^2\alpha_j & \cos\alpha_j \cos\beta_j & \cos\alpha_j \cos\gamma_j \\ \cos\beta_j \cos\alpha_j & \cos^2\beta_j & \cos\beta_j \cos\alpha_j; \\ \cos\gamma_j \cos\alpha_j & \cos\gamma_j \cos\beta_j & \cos^2\gamma_j \end{bmatrix}.$$

2. Identify the structure new joint coordinates $\mathbf{x}_\upsilon$ for the following iteration υ ($\upsilon>0$):

$$\mathbf{x}_\upsilon = \mathbf{x}_{\upsilon-1} + \mathbf{u}_\upsilon. \quad (5)$$

3. Determine the υ-th iteration internal joint forces, corresponding to linear deformation of structural members:

$$S_{j,\upsilon} = \frac{E A_j}{l_{j,\upsilon-1}} \left(l_{j,\upsilon} - l_{j,\upsilon-1} \right), \quad (6)$$

where is material elasticity modulus A_j, l_j are j-th bar area and length, respectively.

4. Calculate direction cosines of structural units. Applying equilibrium equations of nodes find new joint forces $\mathbf{F}_{s,\upsilon}$.

5. Calculate unbalanced (compensating) joint forces given in eqn (2).

6. Create geometric stiffness matrix $[K_{g,\upsilon}]$, which is associated with the changes in the geometry of the space truss. The matrix $[K_{g,\upsilon}]$ is obtained by the assemblage of the element geometric stiffness matrix ([12, 13])

$$[K_{gj,\upsilon}] = \frac{S_{j,\upsilon}}{l_{j,\upsilon}} \begin{bmatrix} [\bar{k}_{j,\upsilon}] & -[\bar{k}_{j,\upsilon}] \\ -[\bar{k}_{j,\upsilon}] & [\bar{k}_{j,\upsilon}] \end{bmatrix} \quad (7)$$

in the global coordinate system, where

$$[\bar{k}_{j,\upsilon}] = \begin{bmatrix} 1-\cos^2\alpha_{j,\upsilon} & -\cos\alpha_{j,\upsilon}\cos\beta_{j,\upsilon} & -\cos\alpha_{j,\upsilon}\cos\gamma_{j,\upsilon} \\ -\cos\beta_{j,\upsilon}\cos\alpha_{j,\upsilon} & 1-\cos^2\beta_{j,\upsilon} & -\cos\beta_{j,\upsilon}\cos\gamma_{j,\upsilon} \\ -\cos\gamma_{j,\upsilon}\cos\alpha_{j,\upsilon} & -\cos\gamma_{j,\upsilon}\cos\beta_{j,\upsilon} & 1-\cos^2\gamma_{j,\upsilon} \end{bmatrix}. \quad (8)$$

7. Create the structural tangent stiffness matrix

$$[K_{t,\upsilon}] = [K_e] + [K_{g,\upsilon}] \quad (9)$$

and calculate its determinant. In case when it yields the negative magnitude, one states the structure to be geometrically unstable and must interrupt calculation. When the determinant magnitude is positive, one must create equilibrium equation (1) for unbalanced joint force

$$[K_t]\mathbf{u}_\upsilon = \mathbf{F}_{c,\upsilon}$$

to correct the obtained joint displacements.

8. Calculate new nodal coordinates, adding the displacements, resulted from unbalanced nodal forces $\mathbf{F}_{c,\upsilon}$ to the ones obtained in the step 2.

9. Repeat the calculations of steps 3–8. Iterations are interrupted when the unbalanced forces $\mathbf{F}_{c,\upsilon}$ are obtained to be infinitesimally small or satisfy prescribed tolerance magnitude.

2.2. Mathematical model of the stress-strain analysis problem

The structure is subjected by known external loading, the areas and physical-geometrical properties of structural units are fixed. The critical stresses for tensile bars are proportional to yielding limit. The critical stresses for compressive bars are determined taking into account buckling behaviour, ie taking into account slenderness of the bars. The truss-type structure stiffness is constrained by limiting its nodal displacements.

The structure prior to plastic failure stress-strain state evaluation problem is provided via the following mathematical model [1, 14]:

find

$$\frac{1}{2}\boldsymbol{\lambda}_1^T [G]\boldsymbol{\lambda}_1 - \frac{1}{2}\boldsymbol{\lambda}_1^T [G]\boldsymbol{\lambda}_2 + \frac{1}{2}\boldsymbol{\lambda}_2^T [G]\boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_1^T \left(\mathbf{S}_e - \mathbf{S}_0\right) + \boldsymbol{\lambda}_2^T \left(-\mathbf{S}_e - \mathbf{S}_{cr}\right) \to \max$$

subject to (10)

$$\boldsymbol{\lambda}_1 \geq \mathbf{0},$$
$$\boldsymbol{\lambda}_2 \geq \mathbf{0}.$$

The quadratic programming problem (10) contains the following values:

$\boldsymbol{\lambda}_1 = \left(\lambda_{1,j}\right)^T$ – vector of Lagrange multipliers of complementarity conditions for tensile bars, reading:

$$\lambda_{1,j}\left(-S_{r,j} - S_{e,j} + S_{0,j}\right) = 0; \quad (11)$$

$S_{r,j}$ – j-th finite element (bar) residual force of the running analysis process;

$S_{0,j}$ – j-th finite element limit axial force (yielding force), $S_{0,j} = \sigma_y A_j$, σ_y – material yield limit, A_j – j-th finite element area, arbitrarily fixed or identified via previous optimization process;

$\boldsymbol{\lambda}_2 = \left(\lambda_{2,j}\right)^T$ – Lagrange multipliers of complementarity conditions for compressive bars, reading:

$$\lambda_{2,j}\left(S_{r,j} + S_{e,j} + S_{cr,j}\right) = 0; \quad (12)$$

$S_{cr,j}$ – j-th finite element limit buckling axial force arbitrarily fixed or identified via previous optimization process;

$[G]$– constant per optimization cycle structure finite element discrete model $(n \times n)$-dimensional influence matrix for residual internal forces, reading:

$$[G] = \left[[B][\overline{K}][R][K_t]^{-1}[R]^T [\overline{K}][B]^T - [\mathcal{K}] \right]. \quad (13)$$

Here:

$[\mathcal{K}]$– tension-compression diagonal stiffness $\frac{E A_j}{l_j}$ matrix of structural elements j = 1, 2, …, n (n – total number of bars), where $E,\ l_j$ are material elasticity modulus and j-th bar length, respectively;

$[K_t]$ – structure finite element $(m \times m)$-dimensional discrete model global tangent stiffness matrix, where m number of global displacements;

$[B]$ – $(n \times 6n)$-dimensional configuration matrix, containing either zero elements or configuration submatrices for internal forces

$$[B_j] = (b_{j,k}),\ k = 6j-5,\ 6j-4,\ ...,\ 6j, \quad (14)$$

expressed via direction cosines of deformed structure in the global coordinate system:

$$[B_j] = (0 \ \ 0 \ \ 0 \ \ \cos\alpha_j \ \ \cos\beta_j \ \ \cos\gamma_j\ ); \quad (15)$$

$[\overline{K}]$– quasidiagonal $(6n \times 6n)$-dimensional matrix, containing j-th space finite element elastic (4) and geometric (7) stiffness diagonal submatrices

$$[\overline{K}_j] = [K_{e,j}] + [K_{gj,\upsilon}]; \quad (16)$$

$[R]$ – $(6n \times m)$-dimensional configuration matrix of local and global displacements, containing unit and zero components.

Solution of the quadratic programming problem (10) yields the Lagrange multipliers λ_1 and λ_2 magnitudes. The components of residual internal forces $\mathbf{S}_r = (S_{r,j})^T$ and that of residual displacements $\mathbf{u}_r = (u_{r,i})^T = (u_{r,1}, u_{r,2}, ..., u_{r,m})^T$ are determined by:

$$\mathbf{S}_r = [G](\lambda_1 - \lambda_2), \quad (17)$$

$$\mathbf{u}_r = [H](\lambda_1 - \lambda_2). \quad (18)$$

Here $$[H] = \left[[K_t]^{-1}\ [R]^T [\overline{K}] [B]^T \right].$$

Note that one cannot state the above obtained residual structural response values to be the exact ones. Having identified above-mentioned values of residual response, the total magnitudes of internal forces and displacements are obtained by:

$$\begin{aligned} \mathbf{S} &= \mathbf{S}_r + \mathbf{S}_e; \\ \mathbf{u} &= \mathbf{u}_r + \mathbf{u}_e. \end{aligned} \quad (19)$$

The member forces resulting at the end of non-linear analysis are employed to obtain the new design variables magnitudes for the next optimization cycle. The minimum weight truss-type structure members cross-sections must adapt to the current parameters of deformable state.

2.3. Mathematical optimization model of structure bars cross-sectional areas

The structure bars cross-sectional areas optimization model under stiffness and stability constraints consists of:

1. Axial plastic strength conditions expressed via areas of optimized bars;
2. Strength conditions versus buckling of bars;
3. Displacements limitations constraints;
4. Constructive limitations for bars areas.

The structure optimality criterion expresses total material minimum weight of the bars.

Thus, the structure optimization model under presence of all above-mentioned conditions is as follows:

find

$$W = \rho \sum_{k=1}^{n_0} A_k \sum_{r=1}^{n_k} l_r \to \min$$

subject to

$$\begin{aligned} & -\sigma_y A_{k,j} \le S_j, \\ & -\sigma_{cr} A_{k,j} \le S_j,\ k = 1, 2, ..., n_0, \\ & u_t = \sum_{j=1}^{n} \frac{S_{t,j} \cdot \overline{S}_{t,j} \cdot l_j}{E A_{k,j}} \le u_{t,\max}, \\ & u_t = \sum_{j=1}^{n} \frac{S_{t,j} \cdot \overline{S}_{t,j} \cdot l_j}{E A_{k,j}} \ge u_{t,\min},\ t = 1, 2, ..., m_t, \\ & A_k \ge A_{k,\min}. \end{aligned} \quad (20)$$

Here:

A_k is the optimized cross-sectional area of the k-group of bars; n_k is the number of bars of k-th group and l_k is the lengths of the bars;

$n_1 + n_2 + ... + n_{n_0} = n$; n_0 is the number of optimized parameters - number of different group of. bars;

ρ is structure material density;

σ_y, σ_{cr} are material yield limit (critical stress for a tension bars) and critical stress of compressive-buckled bars, respectively;

u_t is displacement being constrained along the certain direction t, and m_t is the number of limited displacements;

$u_{t,\max} > 0$, $u_{t,\min} < 0$ are the upper and lower bounds for displacement to be limited;

$A_{k,\min}$ is the lower bound of cross-sectional area magnitude (constructive limitation).

Applying the virtual displacement principle, displacement u_t can be expressed by:

$$u_t = \sum_{j=1}^{n} \frac{S_{t,j} \cdot \bar{S}_{t,j} \cdot l_j}{E\,A_{k,j}} . \tag{21}$$

Here:

$S_{t,j}$ is j-th bar internal force defined by (19), resulting from the elastic-plastic non-linear analysis problem solution;

$\bar{S}_{t,j}$ is j-th bar internal force caused by unit load applied in the direction of the restricted displacement t of the truss, being in the state prior to plastic collapse.

The structural unit design stress versus buckling due EN3 [15] are calculated by:

$$\sigma_{cr} = \chi \sigma_y , \tag{22}$$

where χ is certain bar reduction coefficient, depending on compressive element dimensionless slenderness $\bar{\lambda}$. The dimensionless slenderness is prescribed by ratio:

$$\bar{\lambda} = \lambda / \lambda_E . \tag{23}$$

Here the unit slenderness is defined by $\lambda = l_b / i$, where l_b is an actual length of the buckled member, i is the radius of gyration, $\lambda_E = \sqrt{\dfrac{\pi^2 E}{\sigma_y}}$ is the Euler's slenderness.

For example, the tube cross-sectional reduction coefficient analytically is described by relation as is suggested in [15]:

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}} \le 1 . \tag{24}$$

Here

$$\varphi = \frac{1}{2}\left[1 + \alpha\left(\bar{\lambda}^2 - 0{,}2\right) + \bar{\lambda}^2\right], \tag{25}$$

where α is the variance coefficient (eg $\alpha = 0{,}21$ for hot laminated pipes, that of $\alpha = 0{,}34$ for cold laminated pipes).

The structure bars cross-sectional areas optimization problem is convex non-linear mathematical programming problem with one extremum. Two solution cases due to possible admissible sets of variables are shown in Fig 2 and Fig 3 for two optimized parameters.

Fig 2 presents an optimal solution in the case when the displacement constraints are not activated. The optimization process is preconditioned by strength/stability constraints.

Fig 3 presents the case when the optimal solution is preconditioned also by stiffness constraints. In the considered case the optimal solution contains the satisfied as equality leastwise one strength/stability condition in concert with stiffness constraint. It is obvious that the optimal solution objective function in this case results in a more structural weight magnitude, when compared with the obtained one, presented in Fig 2.

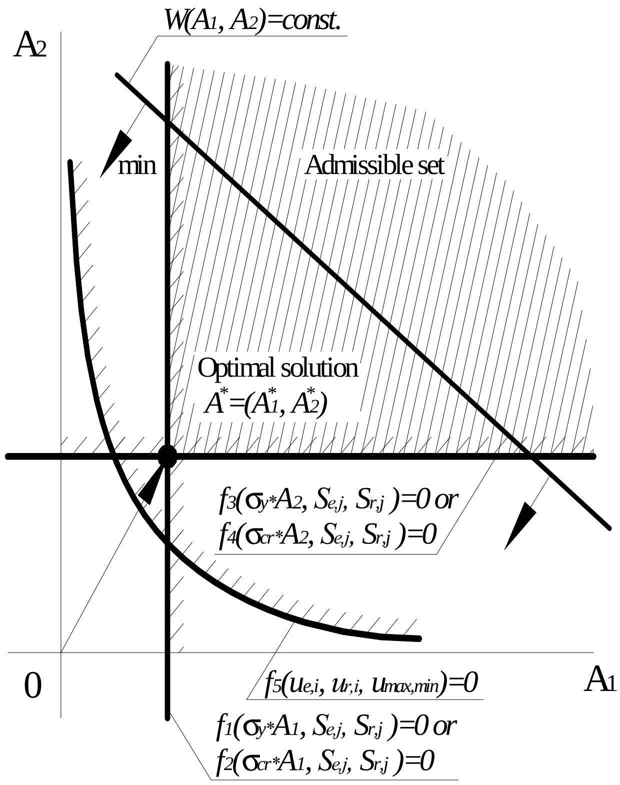

Fig 2. Piecewise-linear admissible set

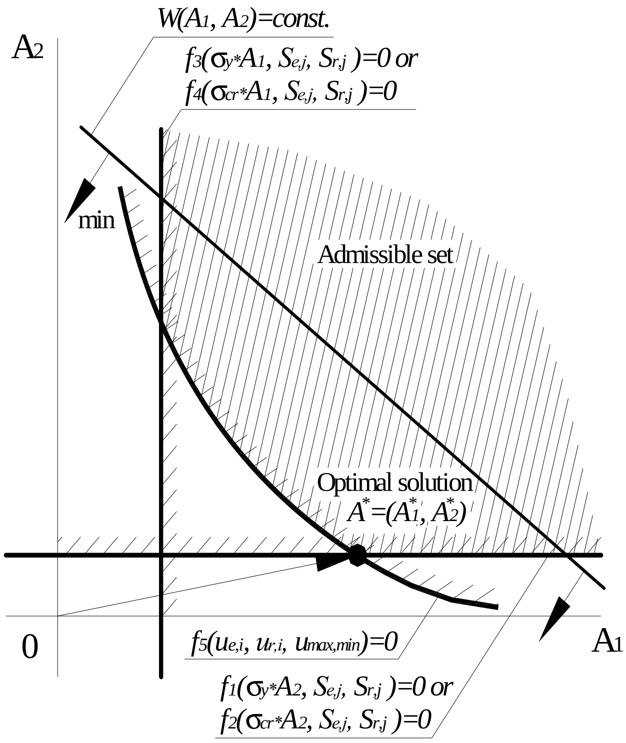

Fig 3. Non-linear admissible set

Analysing the above figures, one can obviously find that the optimization process converges to unique solution for both possible cases, in case when leastwise one truss-type structure member is deformed plastically. When all truss members are elastically deformed, the structural optimization is provided in the elastic range in respect of displacement constraints.

2.4. Analysis and optimum design algorithm main steps

The minimum weight elastic-plastic truss-type structure optimization consists of the following eight steps:

1. Create local and global displacements configuration matrix containing unit or zero components.

2. Determine limit axial forces $\mathbf{S}_0$ and $\mathbf{S}_{cr}$ (Choose primary cross-sectional areas of bars for the start of optimization procedures, introduce the new ones according to the previous iteration optimization result for the current optimization iteration).

3. Create structure discrete model quasidiagonal tangent stiffness matrix $[\overline{K}]$ for tensile-compressive elements (16) and global tangent stiffness matrix $[K_t]$.

4. Determine the force $\mathbf{S}_e$ and displacement $\mathbf{u}_\mathbf{e}$, available at the end of geometric non-linear tangent stiffness analysis.

5. Create influence matrix $[G]$ of residual internal forces.

6. Solve the stress-strain state evaluation quadratic programming problem (10) to find residual response values.

7. Find the Lagrange multipliers $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ magnitudes. Applying the eqns (17)–(19) determine the total magnitudes of internal forces and displacements for the next optimization cycle.

8. Solve structure areas optimization problem (20) under present strength, stiffness and stability constraints.

9. Check the optimization problem prescribed convergence criterion in respect of previous iteration result. When it is not satisfied, repeat steps 2–9. The prescribed criterion can be the fixed admitted tolerance of structure weight function that of the bars areas (both criteria are correlated).

Note that the structure, subjected by external loading, optimization problem solution number of iterations depends on successful starting point (ie chosen for 1-st step primary input data (bars areas)). They can be chosen arbitrarily, solving eg the linear-elastic problem or any similar one, being naturally close to actual constraints of the problem under investigation.

2.5. Numerical examples

The proposed design algorithm is illustrated when applied for design of two different elastic-plastic steel pipe type cold laminated member truss-type structures. Geometrical non-linear deformable behaviour is evaluated.

As for the first example consider the structure of 25 bars (Fig 4) subjected by four external forces, applied to 1, 2 3 ir 6 nodes, respectively. The above forces to structure global axes are as follow:

$$F_{1x} = 120 \text{ kN}, \quad F_{1y} = 30 \text{ kN}, \quad F_{1z} = 80 \text{ kN};$$
$$F_{2x} = 100 \text{ kN}, \quad F_{2y} = 30 \text{ kN}, \quad F_{2z} = 60 \text{ kN};$$
$$F_{3x} = 0, \quad F_{3y} = 0, \quad F_{3z} = 30 \text{ kN};$$
$$F_{6x} = 0, \quad F_{6y} = 0, \quad F_{6z} = 30 \text{ kN}.$$

Eight groups (ie eight different truss areas) of pipe type bars (Fig 4) are optimized to ensure truss minimal weight under strength, stiffness, stability and constructional constraints. The material properties are as follows: yield limit $\sigma_y = 240$ MPa, elasticity modulus E = 207 GPa.

Truss nodes 1 and 2 extreme displacements are constrained in directions x and z by 1 cm. The constructional requirement introduces minimal bars area to be 2 cm^2.

The design history is given in Table 1. As starting point input data (see row wu of Table 1) was taken an optimization solution of the considered truss, when neglecting the dispalacements limiting constraints, loaded by the same four loads. The starting point was obtained in fifteen iterations, applying the same algorithm (20), for its starting point taking all areas to be equal to 10 cm^2 ones (row s of Table 1). One must note that the start problem, neglecting the displacement constraints, is a linear programming one.

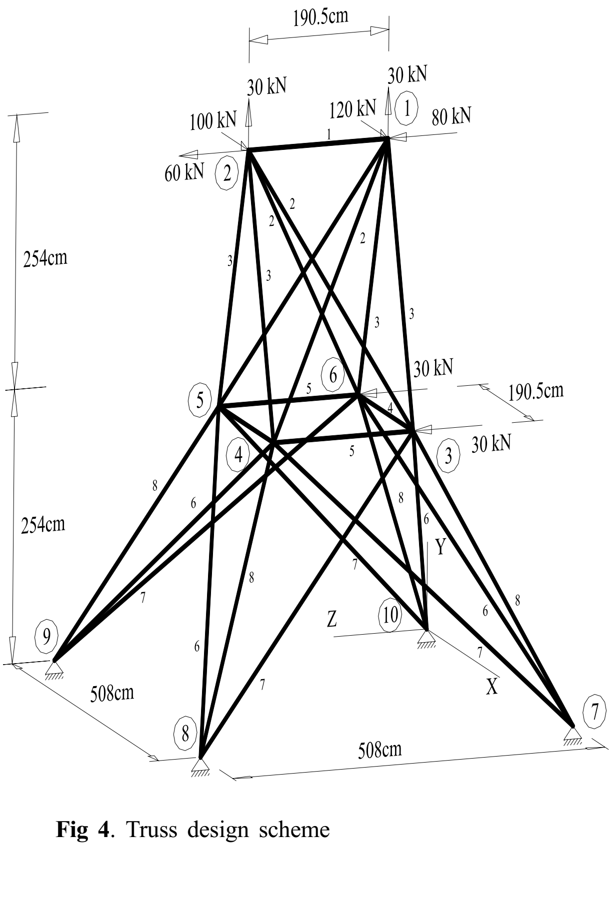

Fig 4. Truss design scheme

Table 1. Optimization problem solutions per iterations

Iteration number	A_1, (cm^2)	A_2, (cm^2)	A_3, (cm^2)	A_4, (cm^2)	A_5, (cm^2)	A_6, (cm^2)	A_7, (cm^2)	A_8, (cm^2)	Truss weight (kg)
s	*10,000*	*10,000*	*10,000*	*10,000*	*10,000*	*10,000*	*10,000*	*10,000*	*659,00*
wu	**2,000**	**8,892**	**6,956**	**2,000**	**2,098**	**5,924**	**3,360**	**11,890**	**427,47**
1	2,000	10,173	10,453	2,000	2,097	5,925	3,635	17,339	532,50
2	2,000	16,207	9,870	2,000	2,737	5,501	3,101	15,592	559,79
3	2,000	11,859	9,240	2,000	2,097	5,577	2,854	18,172	532,25
4	2,000	10,946	9,963	2,000	2,675	5,437	2,768	18,229	527,98
5	2,000	10,090	10,634	2,000	3,251	5,404	2,720	18,229	525,33
6	2,000	9,287	11,264	2,000	3,764	5,390	2,694	18,213	523,14
7	2,000	8,579	11,882	2,000	3,726	5,380	2,696	18,243	520,94
8	2,000	8,025	12,331	2,000	3,992	5,367	2,751	18,167	519,78
9	2,000	8,271	12,488	2,000	4,150	5,359	2,745	17,753	519,54
10	2,000	8,053	12,522	2,000	4,146	5,380	2,758	17,870	519,30
11	2,000	8,198	12,542	2,000	4,179	5,375	2,753	17,738	519,50
12	2,000	**8,189**	**12,542**	**2,000**	**4,163**	**5,375**	**2,754**	**17,717**	**519,48**

The truss optimal solution was reached in twelve iterations (last row of Table 1) with 0,12 % tolerance for 2-nd and 8-th areas, less then 0,5 % for 4-th and 7-th areas and 0 % tolerance for remaining areas and total weight of the structure.

Some notes on optimal truss behaviour being adapted to the considered loading process. The 1-st node reached it allowable 1 cm extreme magnitude, while other displacements do not achieve this magnitude.

The 1-st and 4-th areas reached their minimum equal to 2 cm^2 area magnitudes. The 1–4 and 4–9 truss compression members are loaded up to the critical values – the buckling constraints are dominant ones. Critical states for tensile members are not achieved. A deformable behaviour comparison of the optimized truss with one, obtained when neglecting the displacement constraints (ie with the starting point representing optimization result – row wu of Table 1). When ignoring the truss displacement constraints the solution yields the compression members 2–4, 3–4, 4–7, 4–8 and 4–9 were loaded up to the critical values. Critical states for tensile members were neither achieved. It is interesting to find the optimal solution, obtained when taking into account the dispalcemt limitations yields the 21,5 % more weighted truss. The same truss was design is provided in [6]. One can confirm the complete of optimal solution with the one obtained by proposed method for analogous constraints.

As for the second example, the spherical shell shape hinge-bar structure (Fig 5) is investigated. The truss is subject to a vertical loading of 150 kN at all joints, acting in the negative direction of y-axis. The structure has 21 joints and 52 members are collected into eight different groups. The grouping of members is as shown in the above Figure. The truss members are produced of cold laminated pipes, of thickness 3 mm, elasticity modulus E = 210 Gpa, yield limit σ_y = 240 MPa, density ρ = 7850 kg/m^3.

Having provided the pipe assortment analysis, it was found the ratio of cross-sectional area and radius gyration to be constant one, equal to $A/i = 2{,}65$. Applying the latter relation the pipe profile buckling condition yield the lower bound of cross-sectional area $A_{\min} \geq 2{,}65 l_b / \lambda_{cr}$ (l_b – actual buckling length, λ_{cr} – limit slenderness). The calculations resulted in the following cross-sectional area limits:

First, second and third groups – $A_{\min} = 8{,}0$ cm^2;
Forth, fifth and sixth groups – $A_{\min} = 9{,}0$ cm^2;
Seventh and eighth groups – $A_{\min} = 14{,}0$ cm^2;

The structural optimization is provided in respect of three types of displacement constraints:

1. All nodes displacements are limited to 1 cm;
2. Node 1 displacement is limited to 2 cm, all remaining nodes – to 1 cm;
3. Node 1 displacement is limited to 3 cm, that of for nodes 2–5 to 2 cm, remaining nodes – to 1 cm.

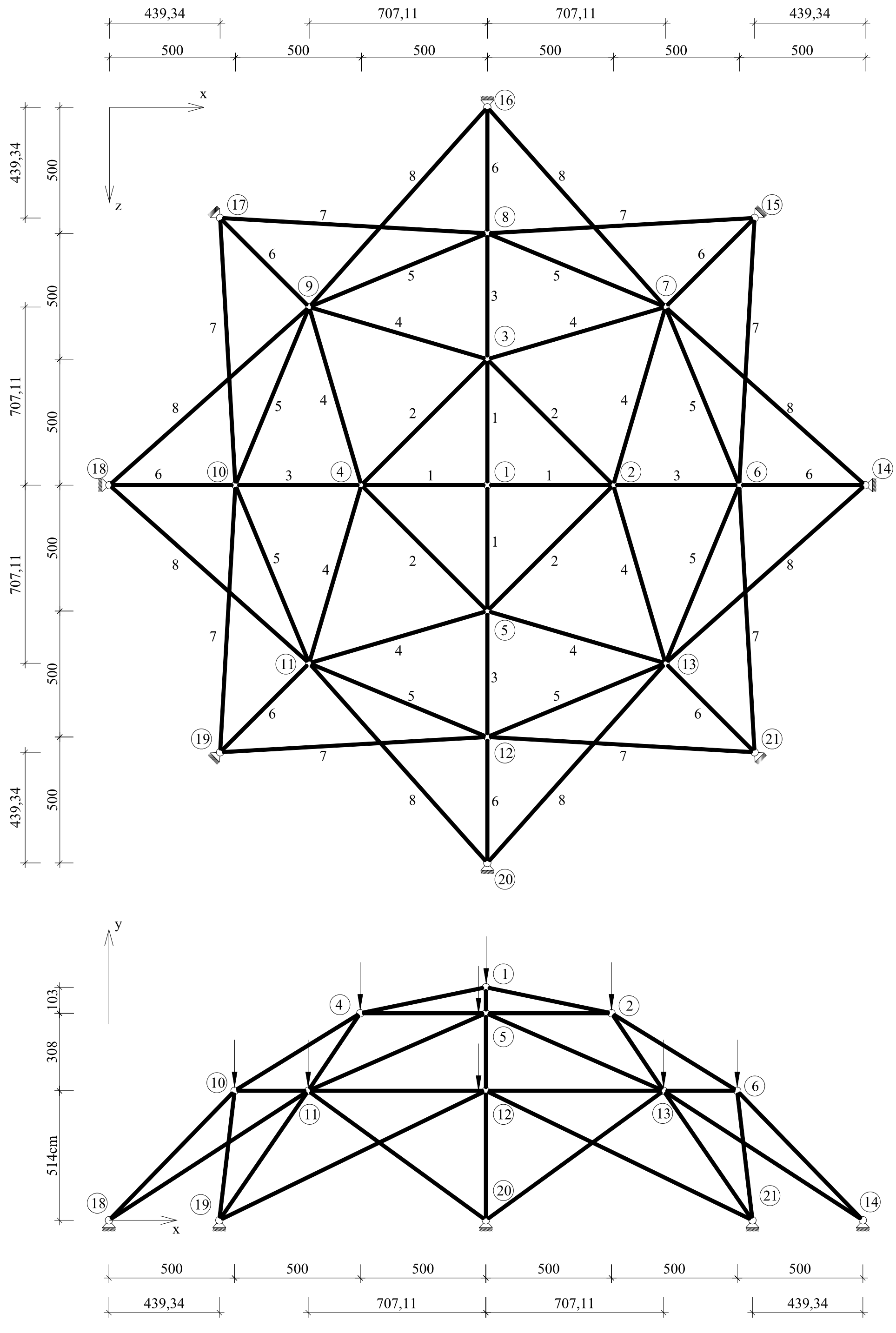

Fig 5. 52-bar space truss

Eight variables are fixed as unknown for the design problem. The initial values of these are chosen to be 20 cm^2 (*s* row of Table 2).

The optimal solution in case of first displacement limiting condition was obtained in five iterations (row W1 Table 2). All the members of the truss are compressed. Note that the displacement constraints predominate in the optimal solution. The displacements reach the limiting magnitudes for all nodes. Any truss member is loaded up to yielding.

The similar view is obtained in case of the second displacement limiting condition. The problem starting point was chosen the solution of the first problem (row W2 of Table 1). The optimal solution was obtained in three iterations. All members are under compression, but nodal displacement reach their limit magnitudes only for 1, 2, 3, 4 ir 5 nodes. The plastic deformations do not appear. Find that both displacement limitations yield elastic behaviour of the structure.

Table 2. The design history of 52-bars truss

Iteration number	A_1, (cm^2)	A_2, (cm^2)	A_3, (cm^2)	A_4, (cm^2)	A_5, (cm^2)	A_6, (cm^2)	A_7, (cm^2)	A_8, (cm^2)	Truss weight (kg)
s	*20,000*	*20,000*	*20,000*	*20,000*	*20,000*	*20,000*	*20,000*	*20,000*	*6963,1*
1	45,258	8,000	35,257	16,301	9,000	27,820	14,000	14,000	6131,5
2	38,764	8,000	32,291	11,963	9,000	28,126	14,000	14,000	5769,0
3	38,525	8,000	32,960	11,478	9,000	28,354	14,000	14,000	5763,4
4	38,389	8,000	33,131	11,371	9,000	28,439	14,000	14,000	5762,8
W1	**38,372**	**8,000**	**33,153**	**11,370**	**9,000**	**28,445**	**14,000**	**14,000**	**5762,7**
1	15,113	9,497	30,666	12,045	9,000	27,285	14,000	14,000	5359,3
2	15,298	9,280	30,542	12,154	9,000	27,135	14,000	14,000	5353,8
W2	**15,292**	**9,238**	**30,497**	**12,193**	**9,000**	**27,116**	**14,000**	**14,000**	**5353,2**
1	15,095	8,000	13,530	9,000	9,000	13,106	14,000	14,000	4218,5
2	14,462	8,000	10,864	10,575	9,000	14,273	14,000	14,000	4290,8
3	15,037	8,000	11,844	9,000	9,000	13,105	14,000	14,000	4186,5
4	14,592	8,000	11,594	9,899	9,000	13,631	14,000	14,000	4242,0
5	14,602	8,000	11,562	9,906	9,000	13,605	14,000	14,000	4242,30
W3	**14,598**	8,000	**12,536**	**9,902**	9,000	**13,594**	14,000	14,000	**4242,20**

The deformational behaviour of truss changes in principle in case of the third displacement limiting condition. The problem starting point was chosen the solution of the second problem (row W2 of Table 2). The optimal solution was obtained in six iterations. All members are under compression, the plastic deformation is reached in members 2–6, 3–8, 4–10 and 5–12. The members 6–14, 7–15, 8–16, 9–17, 10–18, 11–19, 12–20 and 13–21 are close to stability loss, but all the structure is still geometrically stable. The Table illustrates the weight optimal solution convergence dynamics per iterations.

The analysis of obtained solution proves that the proposed optimization method is compatible to the structure actual behaviour.

3. Conclusions

1. The design algorithm is developed for space truss members cross-section optimization, coupling the displacement and combined stress and stability constraints.

2. The new truss structural optimization model is presented. The proposed optimization method principles can be implemented for other types of structures.

3. Created design algorithm splits the structural optimization problem per cycle into three main parts. Each part provides solution of separate problems, employing the solution result obtained in the previous optimization cycle. The optimization cycles are continued up to prescribed tolerance criterion.

4. The overall stability loss has been checked during the optimization process to insure that the obtained optimum design truss is geometrically stable one.

5. The performed numerical experiments illustrates efficiency of proposed algorithm, when applying the proposed optimization under presence stiffness, stress and stability constraints method for the truss, subjected by known external loading.

6. Numerical trusses simultanously have proved that the employment of the tangent stiffness method for the geometric non-linear truss analysis is rather effective one, requiring two-three iterations to reach the redistribution of the unbalancing loads.

7. It is prudent to remark that evaluation of geometrical and material non-linearities in the optimal structural design lead as well as to the closer actual structural behaviour description comparing with an actual one, as to the minimization of the material resources for the optimal solution ensuring a structural reliability.

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