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ANALYSIS OF THE GLOBAL AND LOCAL IMPERFECTION OF STRUCTURAL MEMBERS AND FRAMES

Charlotte MERCIER[1*], Abdelouahab KHELIL[1], Ali KHAMISI[1], Firas AL MAHMOUD[1], Rémi BOISSIERE[1], Alain PAMIES[2]

[1]*University of Lorraine, IJL, UMR CNRS 7198, Nancy, France*
[2]*SFECE, Paris, France*

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Abstract. Stresses of a structure are determined with a first or a second order analysis. The choice of the method is guided by the potential influence of the structure's deformation. In general, considering their low rigidity with regard to those of buildings, scaffolding and shoring structures quickly reach buckling failure. Imperfections, such as structural defects or residual stresses, generate significant second order effects which have to be taken into account. The main challenge is to define these imperfections and to include them appropriately in the calculations. The present study suggests a new approach to define all the structure's imperfections as a unique imperfection, based on the shape of elastic critical buckling mode of the structure. This study proposes a method allowing to determine the equation of the elastic critical buckling mode from the eigenvectors of the second order analysis of the structure. Subsequently, a comparative study of bending moments of different structures calculated according to current Eurocode 3 or 9 methods or according to the new method is performed. The obtained results prove the performance of the proposed method.

Keywords: geometrical imperfections, second order analysis, scaffolding.

Introduction

Thin structures are very prone to buckling failure mainly caused by compression loads. Structures must be analysed using a model and assumptions that reflect their behaviour with an appropriate accuracy in relation to limit states. Stresses in a structure are determined from a first or a second order analysis, according to the potential influence of its deformation (Girão Coelho, Simão, & Wadee, 2013). Usually, it can be considered that scaffolding and shoring structures quickly reach their buckling failure thresholds. Our investigation aims to highlight a calculation method that is both safe and economical.

Application of a compressive force on a member with imperfections generates a bending moment, called second order moment, which leads to greater lateral deformations of this member (Elishakoff, 1978). These deformations amplify the compressive axial force and the lever arm (Shayan, Rasmussen, & Zhang, 2014). The bending moments cause parasitic effects which limit the load capacity of the member (Frey, 2014), explaining the need to take into account accurately imperfections of members in the structure analysis, either in including imperfections in the global analysis, or, in using appropriate criteria taken into account the unmodelled imperfections (Maquoi, Boissonnade, Muzeau, Jaspart, & Villette, 2001; Eindhoven University of Technology, 2006).

Due to the fact that imperfections of construction elements are within the normal tolerances of manufacturing lines, they are generally not visible and cannot be quantified precisely in advance. Several approaches have been conducted to study the imperfections that can reduce the structure resistance (Taheri-Behrooz & Omidi, 2018). Kala (2005) proposed a study to determine the influence of the imperfections on the resistance of members subjected to axial compression. Taheri-Behrooz, Esmaeel, and Taheri (2012), Taheri-Behrooz, Omidi, & Shokrieh (2017) analysed the buckling behaviour of composite cylinders with cutout. They developed finite element analysis, with first or second order approaches, enabling to predict on the buckling behaviour of thin cylinders, the effects of initial imperfections, the effects of the size and the direction of a cutout and the effects of combined initial imperfections and cutout.

*Corresponding author. E-mail: *charlotte.mercier@univ-lorraine.fr*

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Two methods are suggested by Eurocode 3 (European Committee for Standardization [CEN], 2005) to include the structural defects in the model of the structure. The first is the conventional method, based on the definition of a local imperfection and a global imperfection of the structure. The second method is based on the definition of a unique global and local imperfection whose appearance is similar to predominant elastic critical buckling mode of the structure.

Analytical researches have been carried out in the recent years concerning the including of the imperfections in the model by means of a unique global and local imperfection. Gonçalves and Camotim (2005) proposed an approach enabling a better understanding of the concepts involved in Eurocode 3 (CEN, 2005) provisions. Agüero, L. Pallarés, and F. J. Pallarés (2015) also proposed a method to define the unique imperfection to flexural and/or torsional buckling steel structures. Chladný and Štujberová (2013a, 2013b), in their articles on the unique local and global imperfection of a structure subjected to elastic buckling, defined a tool to determine the shape of the buckling mode as an initial unique imperfection. Shayan et al. (2014) studied the number and magnitudes of eigenmodes to use in the structural analyses to determine the initial geometric imperfections of steel frames. Hassan, Salawdeh, and Goggins (2018) proposed in their investigation a comparison of three methods to define geometrical imperfections of structural steel hollow sections under cyclic axial loading.

The present study suggests a new approach to define imperfections as a unique imperfection, using the eigenvectors of the second order analysis of the structure. These eigenvectors enable to put in equation the elastic critical buckling mode of the structure. This alternative method changes the approach to Eurocode 3 (CEN, 2005). This present article also proposes a comparative study of bending moments of different structures calculated according to current Eurocode 3 (CEN, 2005) or 9 (CEN, 2010) methods or according to the new proposed approach.

1. Methods of taking imperfections into account

1.1. Local and global imperfections of structures (conventional Eurocode 3 method)

The conventional method of taking into account imperfections according to Eurocode 3 (CEN, 2005) is translated by the definition of a global imperfection and local (see Figure 1). These imperfections are represented respectively by: a global initial default balance of the structure; and a local deformation in arc. In a structure, the imperfections can be introduced either by calculating the coordinates of the nodes of the structure with imperfections or by means of equivalent loads applied on the structure without imperfections.

Eurocode 3 (CEN, 2005) defines:

– Global initial sway imperfections ϕ:

$$\phi = \phi_0 \cdot \alpha_h \cdot \alpha_m, \tag{1}$$

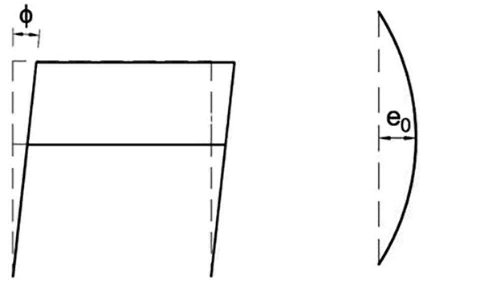


Figure 1. Global and local imperfection

where: $\phi_0 = 1/200$ as the basic value; α_h is the reduction factor for height h applicable to columns: $\alpha_h = \frac{2}{\sqrt{h}}$ but $\frac{2}{3} \le \alpha_h \le 1$; α_m is the reduction coefficient for the number of columns: $\alpha_m = \sqrt{0.5 \cdot \left(1 + \frac{1}{m}\right)}$, where m is the number of columns in a row including only those columns which carry a vertical load not less than 50% of the average value of the column in the vertical plane considered

– Local imperfections of members e_0/L:

e_0 is the bow imperfection for simple columns. Table 1 gives the ratio e_0/l conventionally defined in the European buckling curves (Maquoi & Rondal, 1978) depending on the type of analysis conducted (elastic or plastic).

Table 1. Design value of the initial local bow imperfection e_0/l for members

European buckling curves	Elastic analysis	Plastic analysis
	e_0 / L	e_0 / L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

Curve a_0 corresponds to hot-rolled I-sections with thin flange (at most 40 mm) with steel S460, and hot-rolled hollow sections with steel S460. Curve a represents quasi-perfect shapes such as: hot-rolled I-sections (h/b > 1.2) with thin flange (t_f < 40 mm) if buckling is about major axis, hot-rolled hollow sections (with other steel than S460). Curve b corresponds to shapes with medium imperfections such as: most welded box sections, hot-rolled I-sections (h/b > 1.2) with thin flange (t_f < 40 mm) whose buckling is about minor axis, welded I-sections with thin flanges (t_f < 40 mm) buckling about major axis, L-sections. Curve c represents profiles with a lot of imperfections such as: cold-rolled hollow sections, U-sections, T-sections, solid sections. Curve d corresponds to profiles with maximum imperfections such as: hot-rolled I-sections (h/b < 1.2) with very thick flange (t_f > 100 mm), welded I-sections with thick flange (t_f > 40 mm). Table 6.2 of Eurocode 3 (CEN, 2005) enables to select the buckling curve according to the type of cross-section.

The main fault of these conventional methods is that they rely only on the geometry of the structure. Moreover, it is important to consider the most unfavourable form of these imperfections. Several combinations of global and local imperfections must be considered to find the worst, which is one of the disadvantages of this method.

1.2. Unique local and global imperfection of structures

In terms of structural mechanics, the elastic critical buckling mode (η_{cr}) of any given frame allows anticipating its local and global behaviour. Indeed, the buckling mode expresses the type of deformation that may occur to the right of each node of the structure under the effect of a load. This being assumed, the prior existence of a structural imperfection in the direction of future displacements means that the structure is subjected to the most unfavourable conditions (Baguet, 2001).

In the case where imperfections must be taken into account, a unique global and local imperfection similar to the most representative elastic critical buckling mode will be defined. It is advisable to assign amplitude to this shape that is also representative of the desired level of imperfection. This magnitude can be determined by three methods.

1.2.1. Alternative method of Eurocode 3

This method is based on the determination of a unique imperfection (η_{init}) which allows the local superposition of the critical distortion of the structure that houses the critical cross-section and a deformed reference member whose geometrical characteristics and critical load are the same as those of the critical cross-section of the structure.

The alternative method of Eurocode 3 (CEN, 2005) is based on the following requirement: the curvature in the critical cross-section of the structure with the initial imperfection, based on the elastic critical buckling mode η_{cr}, should be equal to the maximum curvature of the reference member.

The factor for amplifying the maximum curvature of the elastic critical buckling mode is noted C_{nor}. This allows defining the unique global and local imperfection of the whole structure. Indeed, for each node of the structure, the unique imperfection is the product of the amplification factor and the buckling mode (η_{cr}) at each considered node.

The normalization factor is given by:

$$C_{nor} = \frac{\eta_{init}}{\eta_{cr}} = \frac{e''\begin{pmatrix}\text{maximum curvature}\\ \text{of the reference member}\end{pmatrix}}{\eta''\begin{pmatrix}\text{maximum curvature of the}\\ \text{elastic critical buckling mode}\\ \text{of the structure}\end{pmatrix}}, \quad (2)$$

where: e'' is the maximum curvature of the deformed reference member whose geometrical characteristics and critical load are the same as those of the critical cross-section of the structure; and, η'' is the maximum curvature of the dominant elastic critical buckling mode of the structure.

Therefore, it is necessary to determine the maximum curvature of the elastic critical buckling mode of the structure. The calculation software only gives displacements and rotations at different nodes of the structure. Knowing the displacements and rotations of the different nodes of a member, an equation is written to reconstruct the shape of elastic critical buckling mode of the structure and subsequently to determine the curvatures at all the points of the member.

1.2.2. Eurocode 9 method

This method is a derivative of the method developed in Eurocode 3 (CEN, 2005). Its purpose is also to define a unique global and local imperfection (η_{init}) which is broadly modelled on the elastic critical buckling mode and retained locally on the deformation of the reference member.

After defining the reference member, it is assumed that at the maximum bending point of the structure member, the curvature of the shape of the elastic critical buckling mode must be equal to the maximum curvature of the reference member. As the reference member is modelled with the critical load and geometric characteristics of the critical cross-section of the structure, the maximum bending moment at the critical cross-section of the structure must be equal to the maximum bending moment obtained in the reference member.

Thus, Eurocode 9 (CEN, 2010) approach consists in amplifying the maximum second order moment of the most heavy-loaded member of the structure so that it is equal to the maximum second order moment obtained in the reference member. The amplification factor denoted C_{nor} is that which will allow us to define the unique deformation of the whole structure. Finally, the unique imperfection for each node of the structure is the product of the amplification factor by the buckling deformation mode (η_{cr}).

The normalizing factor in the literature is given by:

$$C_{nor} = \frac{\eta_{init}}{\eta_{cr}} = \alpha \cdot \frac{\bar{\lambda} - 0.2}{\bar{\lambda}^2} \cdot \frac{M_{Rk}}{\left(\alpha_{cr} - 1\right) \cdot M^{II}_{\eta_{cr}}}. \quad (3)$$

1.2.3. Origin of similarities in the different definition of the unique imperfection

As mentioned previously, the elastic critical buckling mode gives us the type of lateral displacements that may occur at each node of the structure under the effect of compression loads. The unique imperfection is defined according to the shape of this elastic critical buckling mode amplified by a C_{nor} factor enabling to achieve the desired level of imperfection called η_{init}:

$$\eta_{init} = C_{nor} \cdot \eta_{cr}. \quad (4)$$

Moreover, the desired level of imperfection is based on the behaviour or appearance of a reference member whose geometrical characteristics and critical load are the same as those of the critical cross-section of the structure.

The C_{nor} amplification factor can be defined thanks to the comparison between:

– the maximum curvature of the reference bar and that of the critical cross-section of the structure:

$$C_{nor}=\frac{\eta_{init}}{\eta_{cr}}=\frac{e''\left(\text{Maximum curvature of the reference member}\right)}{\eta''\left(\text{Maximum curvature of the elastic critical buckling mode}\right)};\qquad(5)$$

– the maximum second order bending moment of the reference bar and that of the critical section of the structure:

$$C_{nor}=\frac{\eta_{init}}{\eta_{cr}}=\frac{M^{II}_{\eta_{init}}\left(\text{Maximum second order bending moment of the reference member}\right)}{M^{II}_{\eta_{cr}}\left(\text{Maximum second order bending moment in the structure}\right)};\qquad(6)$$

– the maximum initial deformation of the reference bar and that of the critical section of the structure:

$$C_{nor}=\frac{\eta_{init}}{\eta_{cr}}=\frac{e_0\left(\text{Maximum deformation of the reference member}\right)}{e\left(\text{Maximum deformation of the virtual bar of the structure}\right)}.\qquad(7)$$

Expressions of the bending moment in a member with an initial deformation

In this part, we sought to express the maximum second order bending moment of a simply supported member (see Figure 2) as a function of the maximum initial curvature $e''_{0,\max}$ due to its initial imperfection $e(x)$. Initial imperfection of the member is defined as sinusoidal:

$$e(x)=-e_0\cdot\sin\left(\frac{\pi\cdot x}{L_{cr}}\right).\qquad(8)$$

Consequently:

$$\left|e_{\max}\right|=e_0\cdot\left[\sin\left(\frac{\pi\cdot x}{L_{cr}}\right)\right]_{\max}=e_0 \text{ with } \left[\sin\left(\frac{\pi\cdot x}{L_{cr}}\right)\right]_{\max}=1 \text{ for } x=\frac{L_{cr}}{2}.\qquad(9)$$

Conventionally, in a simply supported member initially assigned of a unique imperfection η_{init}, the application of a compressive load N_{Ed} at its ends creates a maximum second order bending moment given by the expression:

$$M^{II}_{\max}=M^{II}_{e_{\max}}=N_{Ed}\cdot e_0\cdot\frac{\alpha_{cr}}{\alpha_{cr}-1}.\qquad(10)$$

The critical load of this member is defined by the following expression:

$$N_{cr}=\frac{\pi^2 EI}{L_{cr}^2}.\qquad(11)$$

By multiplying both sides by e_0, we obtain:

$$N_{cr}\cdot e_0=\frac{\pi^2 EI}{L_{cr}^2}\cdot e_0.\qquad(12)$$

Then:

$$\frac{N_{cr}}{N_{Ed}}\cdot N_{Ed}\cdot e_0=\frac{\pi^2 EI}{L_{cr}^2}\cdot e_0.\qquad(13)$$

Considering the fact that $\alpha_{cr}=\frac{N_{cr}}{N_{Ed}}$, the previously equation becomes:

$$\alpha_{cr}\cdot N_{Ed}\cdot e_0=\frac{\pi^2 EI}{L_{cr}^2}\cdot e_0.\qquad(14)$$

By multiplying both sides of this equation by $\frac{1}{\alpha_{cr}-1}$, we obtain:

$$N_{Ed}\cdot e_0\cdot\frac{\alpha_{cr}}{\alpha_{cr}-1}=\frac{\pi^2 EI}{L_{cr}^2}\cdot e_0\cdot\frac{1}{\alpha_{cr}-1},\qquad(15)$$

in other words: $$M^{II}_{e_{\max}}=\frac{\pi^2 EI}{L_{cr}^2}\cdot e_0\cdot\frac{1}{\alpha_{cr}-1}.\qquad(16)$$

From Eqn (8), the expression of the curvature of the member is the following:

$$e''(x)=e_0\cdot\frac{\pi^2}{L_{cr}^2}\cdot\sin\left(\frac{\pi\cdot x}{L_{cr}}\right),\qquad(17)$$

thus: $$e''_{\max}=e_0\cdot\frac{\pi^2}{L_{cr}^2}\cdot\left[\sin\left(\frac{\pi\cdot x}{L_{cr}}\right)\right]_{\max}=e_0\cdot\frac{\pi^2}{L_{cr}^2} \text{ with } \left[\sin\left(\frac{\pi\cdot x}{L_{cr}}\right)\right]_{\max}=1 \text{ for } x=\frac{L_{cr}}{2}.\qquad(18)$$

Therefore Eqn (16) becomes:

$$M^{II}_{e_{\max}}=EI\cdot e''_{\max}\cdot\frac{1}{\alpha_{cr}-1}.\qquad(19)$$

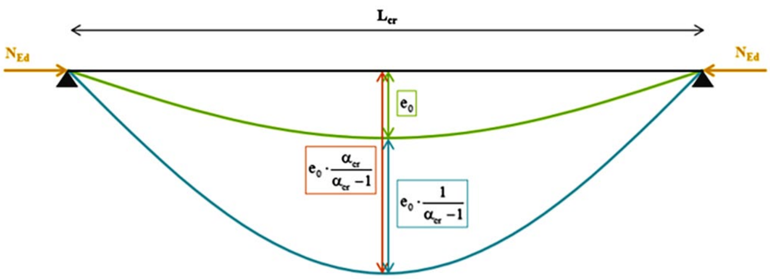


Figure 2. Simply supported member

Finally:

$$M^{II}_{e_{\max}} = \underbrace{N_{Ed} \cdot e_0 \cdot \frac{\alpha_{cr}}{\alpha_{cr}-1}}_{\substack{\text{Moment obtained} \\ \text{manually} \\ (\text{Force} \times \text{Lever arm})}} = \underbrace{EI \cdot e''_{\max} \cdot \frac{1}{\alpha_{cr}-1}}_{\text{Expression of the bending moment as a function of the maximum curvature obtained from numerical calculation of a member assigned of an initial imperfection as same shape as the elastic critical buckling mode}} . \quad (20)$$

Expression of the unique global and local imperfection η_{init}

Now we will attempt to find the expression of the unique imperfection given in the alternative method of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010). This unique global and local imperfection has the same shape as the elastic critical buckling mode. Thus, it possible to write the following ratio:

$$\eta_{init}(x) = \frac{\eta_{init,\max}}{\eta_{cr,\max}} \cdot \eta_{cr}(x), \quad (21)$$

so:

$$\eta''_{init}(x) = \frac{\eta_{init,\max}}{\eta_{cr,\max}} \cdot \eta''_{cr}(x). \quad (22)$$

Taking into account that: $N_{cr} = \alpha_{cr} \cdot N_{Ed}$ and $\bar{\lambda}^2 = \frac{N_{Rk}}{N_{cr}}$, Eqn (20) becomes:

$$\frac{N_{Rk}}{\bar{\lambda}^2} \cdot e_0 = EI \cdot e''_{\max}. \quad (23)$$

As mentioned previously, the alternative method of Eurocode 3 (2005) is based on the following requirement: the curvature in the critical cross-section m of the structure with the initial imperfection, based on the elastic critical buckling mode, should be equal to the maximum curvature of the reference member.
Hence:

$$\eta''_{init,m} = \frac{e_0}{\bar{\lambda}^2} \cdot \frac{N_{Rk}}{EI}. \quad (24)$$

According to Eqn (22), the curvature in the critical cross-section m of the structure with the initial imperfection can be expressed as follows:

$$\eta''_{init,m} = \frac{\eta_{init,\max}}{\eta_{cr,\max}} \cdot \eta''_{cr,m}, \quad (25)$$

therefore Eqn (24) becomes:

$$\eta_{init,\max} = \frac{e_0}{\bar{\lambda}^2} \cdot \frac{N_{Rk}}{EI\eta''_{cr,m}} \cdot \eta_{cr,\max}. \quad (26)$$

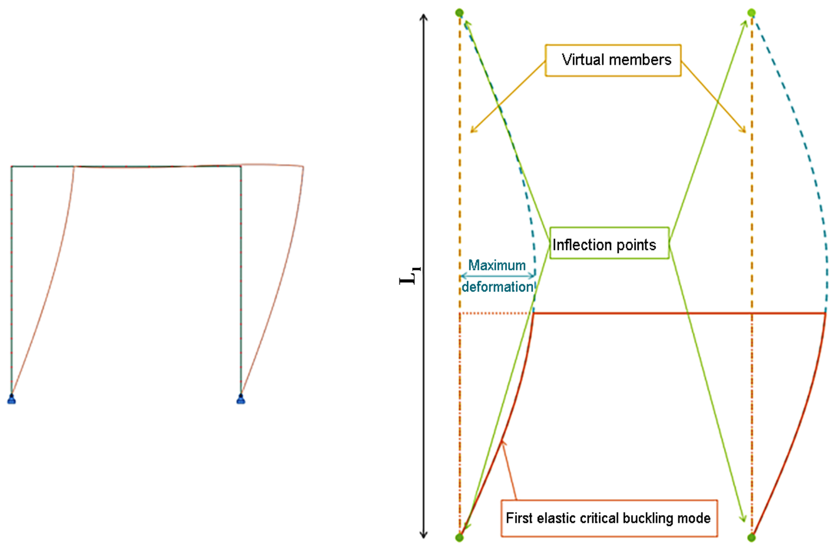
Thus we find the expression of the unique imperfection defined in Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010):

$$\eta_{init}(x) = \frac{e_0}{\bar{\lambda}^2} \cdot \frac{N_{Rk}}{EI\eta''_{cr,m}} \cdot \eta_{cr}(x). \quad (27)$$

2. New approach to define the unique initial imperfection

2.1. Principle

Alternative method of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) consists in comparing the maximum bending moment of the most heavy-loaded section of the structure to the maximum bending moment of the reference member. We proposed studying a pinned-base frame, of 4 m high and 4 m wide, with a cold formed tubular section 60×6 mm, whose cross-sectional characteristics



(a) First elastic critical buckling mode of the structure

(b) Mathematical construction of the virtual member

Figure 3. Construction of the virtual member

are: A = 10.18×10^{-4} m^2, I = 37.56×10^{-8} m^4, W_{el} = 12.52×10^{-6} m^3. The material of the structure is S320: f_y = 320 MPa.

A virtual member is defined as a simply supported member with a height equal to the buckling length of the critical cross-section of the studied structure and with an initial imperfection as same shape as the shape of the most relevant elastic critical buckling mode. The virtual member has the same behaviour as the element of the structure when it is subjected to a compressive load equal to the critical load of the critical cross-section of the structure. It was observed, that when the length of the virtual member is defined as being the length between two successive inflection points of the shape of the elastic critical buckling mode of the structure (see Figure 3), the maximum bending point of the structure does not always correspond to the point where the derivative of the curve is equal to zero.

Indeed, in the case of a pinned-base frame the maximum bending moment, and so the maximum curvature, is obtained in the cross-section located at Z = 4.00 m. However in the reference member the maximum bending moment and the maximum curvature are obtained in the mid-height cross-section, that is to say at Z = 4.65 m. The calculation of the amplification factor according to Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) approaches is performed by comparing two points that are not located at the same position. There are other cases of structures where the gap these two points is expressed even more strongly. This gap can lead to an error in the calculation of the amplification factor and so in the calculation of the initial imperfection of the structure (Figure 4).

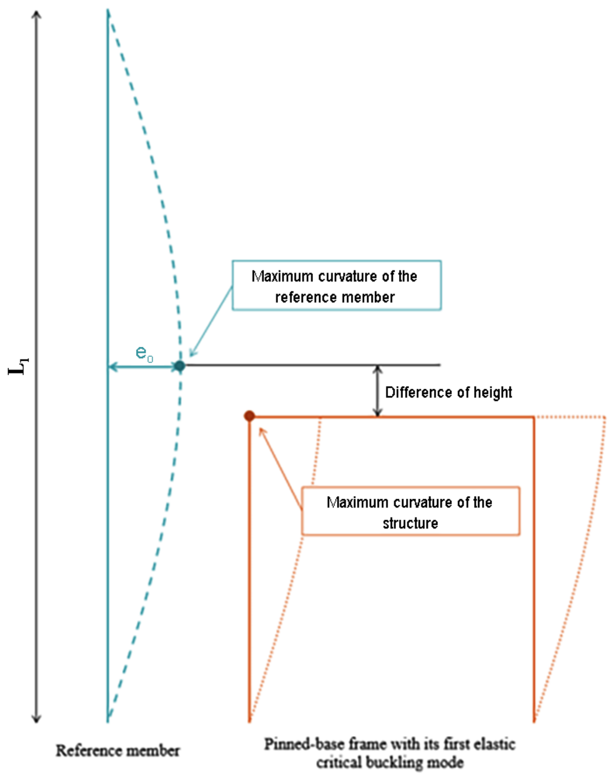


Figure 4. Position of the maximum curvature in the pinned-base frame and in its reference member

To overcome this problem we propose a new approach consists in establishing a mathematical equation of the shape of the elastic critical buckling mode of the structure. The sine term of the equation of the shape of the critical buckling mode of the structure allows us to mathematically reconstruct it and set its inflection points. These inflection points allow building a virtual member from which the "imaginary" maximum deformation and the "imaginary" maximum curvature of the structure are deduced. This maximum curvature (or maximum deformation) will be compared to the maximum curvature (or maximum deformation) of the reference member. The factor C_{nor} will then be used to amplify the deformation of buckling mode (η_{cr}) and then to obtain the desired unique imperfection.

The equation of the shape of the predominant elastic critical buckling mode η_{cr} of the structure is:

$$\eta_{cr}(z) = A \cdot \cos\left(\frac{\pi \cdot z}{l_f}\right) + B \cdot \sin\left(\frac{\pi \cdot z}{l_f}\right) + C \cdot z + D. \quad (28)$$

From the eigenvectors of the predominant elastic critical buckling mode of only two nodes of the structure (called node1 and node 2), it is possible to set up and solve the following linear system of equations and thus determine the unknown A, B, C and D.

$$\begin{cases} A \cdot \left[\cos\left(\frac{\pi \cdot l}{l_f}\right) - 1\right] + B \cdot \left[\sin\left(\frac{\pi \cdot l}{l_f}\right) - \frac{\pi \cdot l}{l_f}\right] = \Delta - RY_1 \cdot l \\ A \cdot \left[-\sin\left(\frac{\pi \cdot l}{l_f}\right)\right] + B \cdot \left[\cos\left(\frac{\pi \cdot l}{l_f}\right) - 1\right] = \frac{RY_2 - RY_1}{\pi} \cdot l_f \end{cases}$$

$$with \begin{cases} l = z_1 - z_2 \\ \Delta = UX_2 - UX_1 \end{cases}, \quad (29)$$

where: UX_1, UX_2, RY_1 and RY_2 are the eigenvectors of the both nodes 1 and 2 of the predominant elastic critical buckling mode of the structure.

The expression of the curvature at each point of the structure can be obtained by expressing the second derivative of Eqn (28):

$$\eta''_{cr,\max} = -A \cdot \left(\frac{\pi}{l_f}\right)^2 \cdot \cos\left(\frac{\pi \cdot z_{\eta''_{cr,\max}}}{l_f}\right) - B \cdot \left(\frac{\pi}{l_f}\right)^2 \cdot \sin\left(\frac{\pi \cdot z_{\eta''_{cr,\max}}}{l_f}\right). \quad (30)$$

Furthermore, the position of the maximum curvature point can be determined mathematically from the derivative of the curvature. Thus, the following expression enable to determine the position of the maximum curvature of the predominant elastic critical buckling mode:

$$z_{\eta''_{cr,\max}} = l_f + \frac{\pi}{l_f} \cdot \arctan\left(\frac{B}{A}\right). \quad (31)$$

Finally, $z_{\eta''_{cr,\max}}$ is used to determine the "imaginary" maximum curvature and the "imaginary" maximum deformation of the most heavy-loaded section of the structure. As the latter is located halfway up from the buckling length, it is compared to the maximum curvature or the maximum deformation of the reference member, respectively.

2.2. Application to the pinned-base frame

The new approach is applied to the pinned-base frame, whose cross-sectional characteristics are given in part 3.1., in order to assess its accuracy. Results of other type of structures are given in part 4. The portal frame is loaded so as to obtain a critical coefficient α_{cr} of 2.

To establish the equation of the shape of the elastic critical buckling mode, it's necessary to use two eigenvectors. The displacements and rotations of the both following nodes (Table 2) are used.

Table 2. Eigenvectors of the pinned-base frame

Node	*UX* (m)	*RY* (rad)	*Z* (m)
8	0.97748850	0.104067513	3.5
9	1	0.075917701	4

From the second order analysis of the structure, the buckling length is determined as equal to 9.312 m. From the system of Eqn (29), we determine the value of the unknowns, see Table 3:

Table 3. Values of the unknowns

$A = -4.031\times10^{-9}$	$B = 1.025$
$C = 1.816\times10^{-8}$	$D = 4.031\times10^{-9}$

Then, thanks to Eqn (31), we determine the position of the maximum curvature point. We obtain: $z_{\max} = 4.656$ m. From this value and Eqn (30), we determine the value of the "imaginary" maximum curvature. We obtain $X''_{\max} = 1.167\times10^{-1}\,\text{m}^{-1}$.

These values can be read graphically in Figure 5.

On the previous figure, the shape of the elastic critical buckling mode of the structure and the theoretical deformation of the virtual member, obtained from mathematical Eqns (28)–(31), are superimposed. This overlap enables us to check the physical impact of our approach and allows to identify the critical section which may optionally be real or fictitious, that is to say outside the structure, like the example of the pinned-base structure. Moreover, it also allows comparing the reference member and virtual member.

On the other hand, we determine the maximum curvature of the reference member. This maximum curvature is determined from Eqn (18), with:

$$e_0 = \alpha \cdot \left(\bar{\lambda} - 0.2\right) \cdot \frac{W_{el}}{A}. \qquad (32)$$

From this value of the maximum curvature of the reference member and the imaginary maximum curvature of the structure, we determine the factor C_{nor} from Eqn (5). We obtain $C_{nor} = 1.864\times10^{-2}$. Thus, this factor is determined from the comparison of two points located in the same cross-section in the both members. Thanks to the factor C_{nor} and the shape of the elastic critical buckling mode, we determine the coordinates of the initial imperfection at each nodes of the structure from Eqn (4). The coordinates obtained are given in the Table 4.

3. Numerical examples

3.1. Presentation of the comparative study

A comparative study of the different methods is carried out in order to appreciate the impact of our new method. It concerns the previous pinned-base frame as well

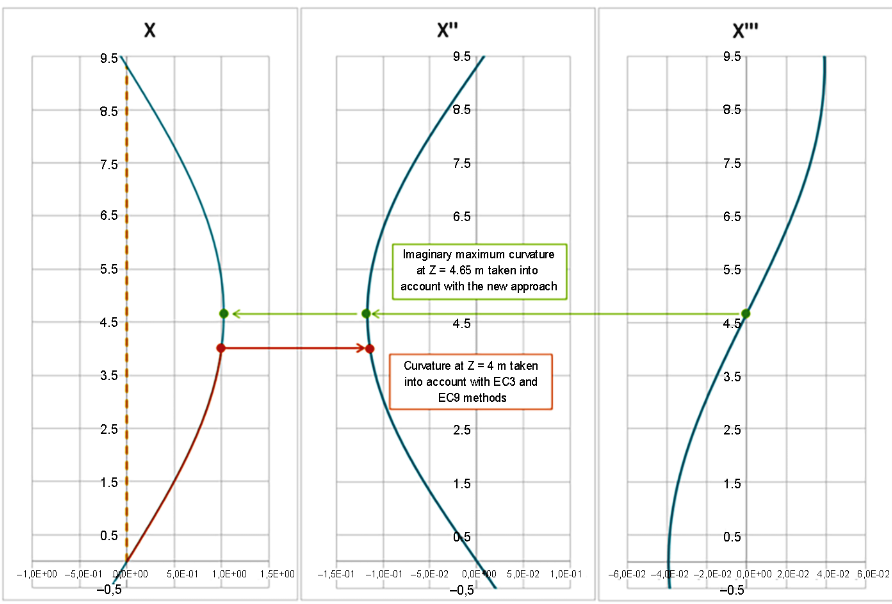


Figure 5. Determination of the imaginary maximum curvature

Table 4. Determination of the coordinates of the nodes of the structure with the initial imperfection

Node	Initial coordinates		Displacement UX of the first buckling mode (m)	Displacements UX of the initial imperfection (m)	Coordinates with the initial imperfections	
	X (m)	Z (m)			X (m)	Z (m)
1	0	0	0	0	0	0
2	0	0.5	0.172080806	0.003207377	0.003207377	0.5
3	0	1	0.339276929	0.006323709	0.006323709	1
4	0	1.5	0.496842343	0.009260536	0.009260536	1.5
5	0	2	0.640304397	0.011934494	0.011934494	2
6	0	2.5	0.765590780	0.014269680	0.014269680	2.5
7	0	3	0.859145116	0.016199806	0.016199806	3
8	0	3.5	0.948027811	0.017670086	0.017670086	3.5
9	0	4	1	0.018638782	0.018638782	4
10	4	0	0	0	4	0
11	4	0.5	0.172080806	0.003207377	4.003207377	0.5
12	4	1	0.339276929	0.006323709	4.006323709	1
13	4	1.5	0.496842343	0.009260536	4.009260536	1.5
14	4	2	0.640304397	0.011934494	4.011924494	2
15	4	2.5	0.765590780	0.014269680	4.014269680	2.5
16	4	3	0.869145116	0.016199806	4.015199806	3
17	4	3.5	0.948027911	0.017670086	4.017670086	3.5
18	4	4	1	0.018638782	4.018328782	4

as three other structures (Figures 6(a)–6(e)): a fixed-base frame of 4 m high and 4 m wide; a shoring tower of 8 m high and 3 m wide; and a façade scaffold of 8 m high and 1.5 m wide.

The characteristics of the different cross-sections are given in the Table 5. Loads on the structure are applied vertically at the top of each column such that the critical coefficient is equal to 1.5, 2 or 5.

The purpose of the study is to compare the different methods of definition the initial imperfections to the new approach, using the virtual member. Five approaches will be compared:

- Conventional method of Eurocode 3 (CEN, 2005), a global imperfection and a local, by calculating the coordinates of the nodes of the structure.
- Conventional method of Eurocode 3 (CEN, 2005), a global imperfection and a local, by using equivalent loads.
- Alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010), a unique global and local imperfection, by calculating the amplification factor Cnor by means of the curvatures.
- Alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010), a unique global and local imperfection, by calculating the amplification factor Cnor by means of the bending moments.
- New approach by using the virtual member in the determining of the proportional factor C_{nor}.

For that we must compare the bending moments obtained in each structure with the different cases of definition the initial imperfection. Calculations were performed using Autodesk Robot Structural Analysis.

3.2. Results and observations from numerical examples

The results obtained with the different structures are presented in Tables 6–10. Figures 7 to 10 enable to compare the calculation point with the new approach and the calculation point with Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) methods.

Table 5. Cross-sectional characteristics of all structures

Portal frames		Scaffold towers		Portal frame in IPE	
Section	60×6 mm	Section	48.3×2.9 mm	Section	IPE100
A (m^2)	10.18×10^{-4}	A (m^2)	4.136×10^{-4}	A (m^2)	10.3×10^{-4}
I (m^4)	37.56×10^{-8}	I (m^4)	10.7×10^{-8}	I (m^4)	171×10^{-8}
W_{el} (m^3)	12.52×10^{-6}	W_{el} (m^3)	4.43×10^{-6}	W_{el} (m^3)	34.2×10^{-6}
f_y (MPa)	320	f_y (MPa)	320	f_y (MPa)	235

3.2.1. Pinned-base frame 4×4 m

Pinned-base frame 4×4 m was presented in Figure 6(a). Figure 5 compares the calculation point with the new approach and the calculation point with Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) methods of the pinned-base frame 4×4 m.

The results obtained with conventional methods of Eurocode 3 (CEN, 2005) are underestimated compared to the results obtained with the new approach. The error with conventional methods can be up to 25%.

It should also be noted that the calculation point of alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) is not at the maximum point of curvature of the virtual member. Thus, there is also an error with these methods up to 6%.

3.2.2. Fixed-base frame 4×4 m

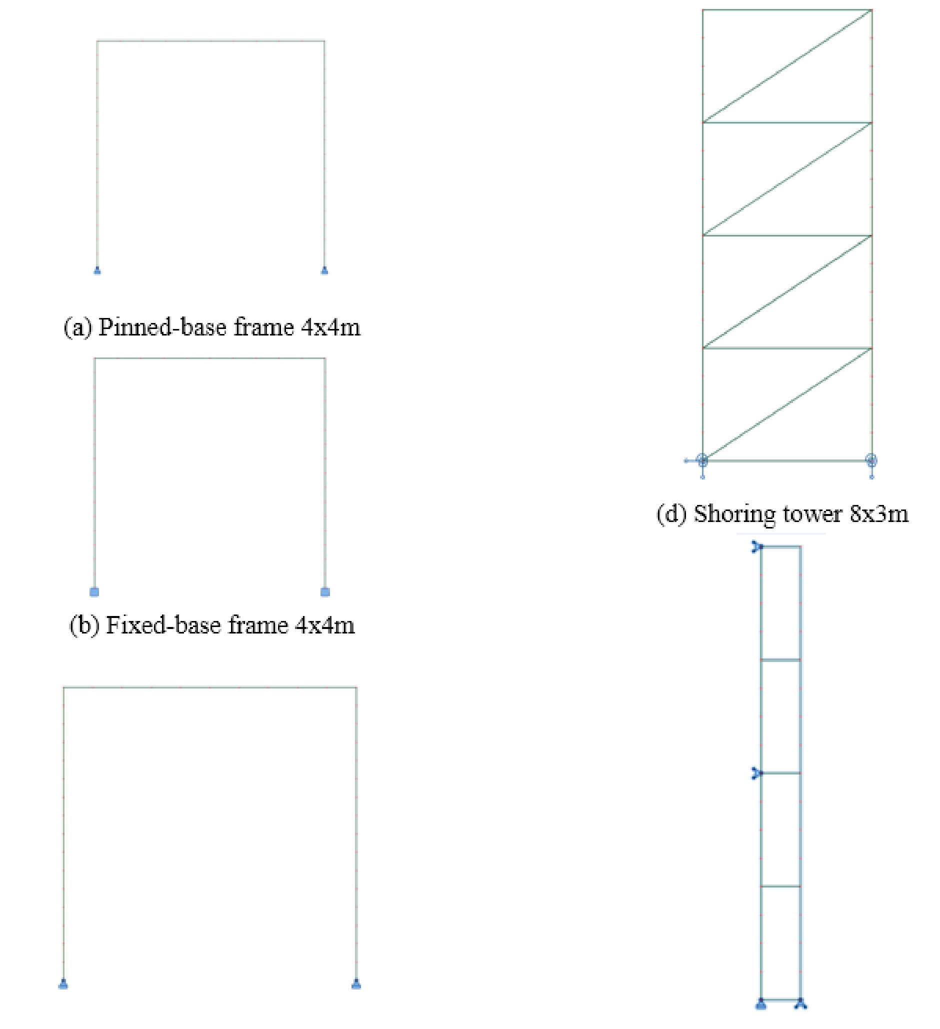
Fixed-base frame 4×4 m was presented in Figure 6(b). Calculation points of the fixed-base frame 4×4 m are depicted in Figure 7.

The results obtained with conventional methods of Eurocode 3 (CEN, 2005) are underestimated compared to the results obtained with the new approach, except in the case of α_{cr} = 5, where the results are overestimated. The error with conventional methods can be up to 40%.

It is important to note that only in the case of the fixed-base frame the calculation point of alternative methods of Eurocode 3 and 9 is the same that the maximum point of curvature of the virtual member. This explains the small error observed with the results obtained from alternative Eurocode 3 and 9 method with the bending moments. On the other hand, as far as alternative Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) method with curvatures is concerned, the error can go up to 18%. Indeed, no way to calculate the curvature is indicated in Eurocode 3 or 9, so depending on the type of equation used (polynomial, sinusoidal, etc.), the error can be more or less important.

3.2.3. Pinned-base frame 5×5 m in IPE

Pinned-base frame 5×5 m in IPE was presented in Figure 6(c). Figure 8 shows calculation points.



(a) Pinned-base frame 4x4m

(b) Fixed-base frame 4x4m

(c) Pinned-base frame 5x5m in IPE

(d) Shoring tower 8x3m

(e) Façade scaffold 8x0.7m

Figure 6. Studied structures

Table 6. Results of the pinned-base frame 4×4 m

	Alternative EC3 and EC9 methods		Conventional EC3 method		New approach
	Curvatures	Bending moments	Manual deformation	Equivalent loads	
Z^* (m)	4	4	–	–	4.656
C_{nor}	0.0322	0.0347	–	–	0.0342
$\alpha_{cr} = 1.5$					
M^{II}_{Ed} (kN.m)	0.5800	0.6259	0.4619	0.4720	0.6167
% Gap	–6%	+1.5%	–25.1%	–23.5%	0%
$\alpha_{cr} = 2$					
M^{II}_{Ed} (kN.m)	0.2897	0.3138	0.2353	0.2387	0.3080
% Gap	–5.9%	+1.9%	–23.6%	–22.5%	0%
$\alpha_{cr} = 5$					
M^{II}_{Ed} (kN.m)	0.0724	0.0786	0.0630	0.0632	0.0769
% Gap	–5.9%	+2.2%	–18%	–17.8%	0%

Note: Z^* – altitude of the comparison point.

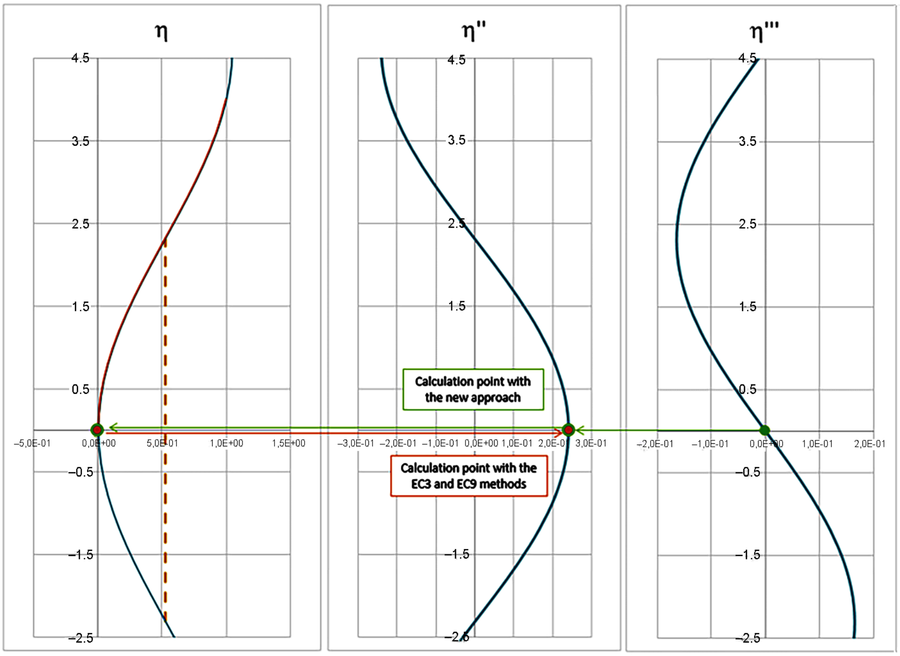


Figure 7. Calculation points of the fixed-base frame 4×4 m

The results obtained with conventional methods of Eurocode 3 (CEN, 2005) are underestimated compared to the results obtained with the new approach. The error with conventional methods can be up to 13%.

It should also be noted, one more time, that the calculation point of alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) is not at the maximum point of curvature of the virtual member. Thus, the error made with these methods can go, in these cases, up to 13%.

3.2.4. Shoring tower 8×3 m

Shoring tower 8×3 m was presented in Figure 6(d). Figure 9 shows calculation points.

In the case of the shoring tower, the results obtained with conventional methods of Eurocode 3 (CEN, 2005) are overestimated compared to the results obtained with the new approach. The error with conventional methods can be up to 50%.

The calculation point of alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) is close to the maximum point of curvature of the virtual member, which explains the small errors observed with alternative Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) methods.

Table 7. Results of the fixed-base frame 4×4 m

	Alternative EC3 and EC9 methods		Conventional EC3 method		New approach
	Curvatures	Bending moments	Manual deformation	Equivalent loads	
Z^* (m)	0	0	–	–	0
C_{nor}	0.0264	0.0316	–	–	0.0322
$\alpha_{cr} = 1.5$					
M^{II}_{Ed} (kN.m)	1.0099	1.2094	0.7467	0.7469	1.2300
% Gap	–17.9%	–1.7%	–39.3%	–39.3%	0%
$\alpha_{cr} = 2$					
M^{II}_{Ed} (kN.m)	0.5037	0.6073	0.4771	0.4816	0.6139
% Gap	–18%	–1.1%	–22.3%	–21.6%	0%
$\alpha_{cr} = 5$					
M^{II}_{Ed} (kN.m)	0.1260	0.1524	0.1626	0.1655	0.1534
% Gap	–17.9%	–0.7%	+6%	+7.9%	0%

Note: Z^*– altitude of the comparison point.

Table 8. Results of the pinned-base frame 5×5 m in IPE100

	Alternative EC3 and EC9 methods		Conventional EC3 method		New approach
	Curvatures	Bending moments	Manual deformation	Equivalent loads	
Z^* (m)	5	5	–	–	5.821
C_{nor}	0.028	0.032	–	–	0.031
$\alpha_{cr} = 1.5$					
M^{II}_{Ed} (kN.m)	1.4399	1.6683	1.4304	1.4723	1.6401
% Gap	–12.2%	+1.7%	–12.8%	–10.2%	0%
$\alpha_{cr} = 2$					
M^{II}_{Ed} (kN.m)	0.7195	0.8363	0.7231	0.7403	0.8197
% Gap	–12.2%	+2%	–11.8%	–9.7%	0%
$\alpha_{cr} = 5$					
M^{II}_{Ed} (kN.m)	0.1798	0.2094	0.1913	0.1940	0.2048
% Gap	–12.2%	+2.3%	–6.6%	–5.3%	0%

Note: Z^*– altitude of the comparison point.

3.2.5. Façade scaffold 8×0.7 m

Façade scaffold 8×0.7 m was presented in Figure 6(e). Figure 10 shows calculation points.

The results obtained with conventional methods of Eurocode 3 (CEN, 2005) overestimated compared to the results obtained with the new approach. The error with conventional methods can be up to 20%.

It should also be noted, one more time, that the calculation point of alternative methods of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) is not at the maximum point of curvature of the virtual member. Thus, the error made with these methods can go, in the case of the façade scaffold, up to 5%.

3.2.6. Conclusion of the numerical examples

Through these results, it can be observed that depending on whether the first buckling mode of the structure is in fixed or sway mode, the classical Eurocode 3 method compared to methods derived from the unique imperfection is safe in the first case and underestimated in the other cases (Tables 6–10).

For structures whose the first buckling mode is a sway mode, the definition of the unique imperfection provides safe results almost identical to those of the current alternative Eurocode 3 (CEN, 2005) method when the buckling length is close to the actual length of the member. However for structures whose the buckling length is superior to the actual length of the member, the determination of the

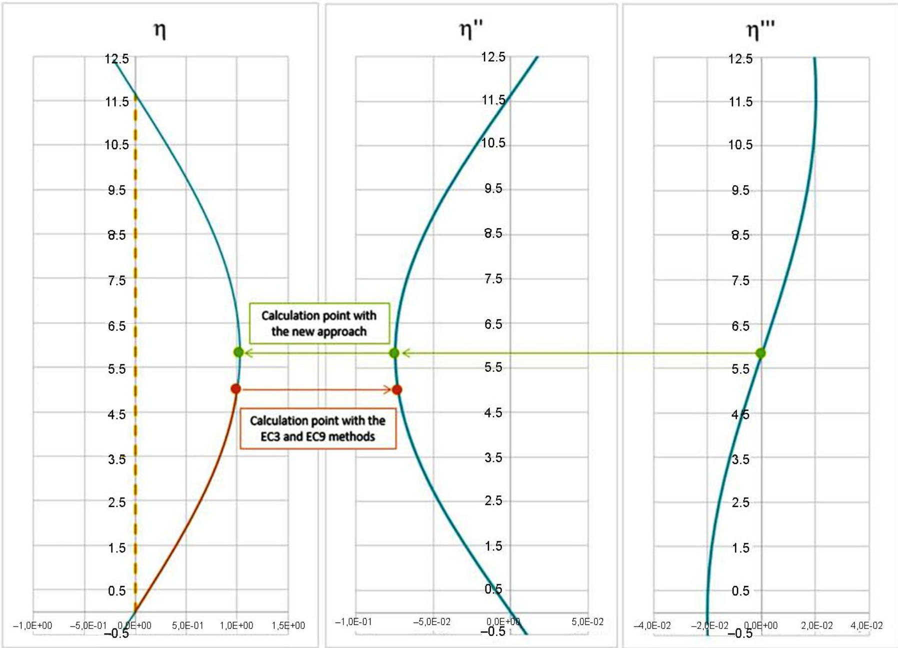


Figure 8. Calculation points of the pinned-base frame 5×5 m in IPE

Table 9. Results of the shoring tower 8×3 m

	Alternative EC3 and EC9 methods		Conventional EC3 method		New approach
	Curvatures	Bending moments	Manual deformation	Equivalent loads	
Z^* (m)	5	5	–	–	5.021
C_{nor}	0.011	0.011	–	–	0.011
$\alpha_{cr} = 1.5$					
M^{II}_{Ed} (kN.m)	0.8522	0.8011	1.2545	0.9360	0.8381
% Gap	+1.7%	–4.4%	+49.7%	+11.7%	0%
$\alpha_{cr} = 2$					
M^{II}_{Ed} (kN.m)	0.4303	0.4187	0.4933	0.4928	0.4233
% Gap	+1.7%	–1.1%	+16.5%	+16.4%	0%
$\alpha_{cr} = 5$					
M^{II}_{Ed} (kN.m)	0.1084	0.1080	0.1344	0.1314	0.1078
% Gap	+0.6%	+0.2%	+24.7%	+21.9%	0%

Note: Z^* – altitude of the comparison point.

amplification factor according to the alternative method of Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010) may lead to errors ranging from –18% to +5% in the present cases but can lead to larger errors in other cases.

During the application of each method, it can also be seen that those based on the definition of a single defect follow the mechanical behaviour of the structure more closely. They not only take into account the geometrical aspects, but also the boundary conditions of the bars, the intrinsic mechanical properties and the load level of the structure. As for the current method of applying local and global imperfection, it leads to ambiguity when taking into account local imperfection in the bars. Moreover, this method is mainly based on geometry and does not take account of the specific properties of the bars.

Conclusions

Our study focused on the development of a method that takes into account structural imperfections in the calculation of scaffolding and shoring structures, since the latter have low critical loads and critical coefficients which are

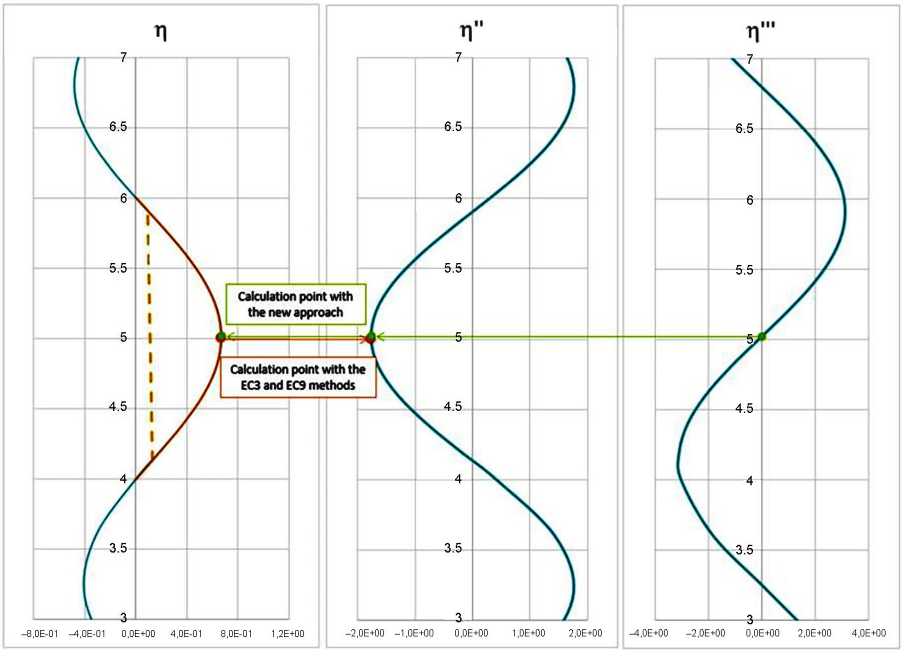


Figure 9. Calculation points of the shoring tower 8×3 m

Table 10. Results of the façade scaffold 8×0.7 m

	Alternative EC3 and EC9 methods		Conventional EC3 method		New approach
	Curvatures	Bending moments	Manual deformation	Equivalent loads	
Z^* (m)	6	6	–	–	5.897
C_{nor}	0.0173	0.0159	–	–	0.0165
$\alpha_{cr} = 1.5$					
M^{II}_{Ed} (kN.m)	0.6683	0.6105	0.7642	0.7546	0.6372
% Gap	+4.9%	–4.2%	+19.9%	+18.4%	0%
$\alpha_{cr} = 2$					
M^{II}_{Ed} (kN.m)	0.3334	0.3058	0.3752	0.3662	0.3180
% Gap	+4.8%	–3.8%	+18%	+15.2%	0%
$\alpha_{cr} = 5$					
M^{II}_{Ed} (kN.m)	0.0835	0.0769	0.0917	0.0885	0.0797
% Gap	+4.8%	–3.5%	+15.1%	+11%	0%

Note: Z^* – altitude of the comparison point.

very vulnerable to second order effects. The objective was to formulate a calculation method that is both safe and economical.

The conventional methods of Eurocode 3 (CEN, 2005), with a global and a local imperfection are based only on the geometry of the structure. It was found that the method based on a unique imperfection has the advantage to be simple and clear, easy to apply and derived from realistic mechanical reasoning. However, in Eurocode 3 (CEN, 2005) and Eurocode 9 (CEN, 2010), the way to determine the maximum curvature of the shape of the elastic critical buckling mode is not given explicitly. The new approach proposed in this article aims to propose a method enabling the determination of this maximum curvature. Following the structure concerned this maximum curvature can be real or imaginary. In all cases, this point of maximum curvature in the structure is the same as the point of maximum curvature in the reference member. This method makes it possible to change the approach to Eurocode 3 (CEN, 2005).

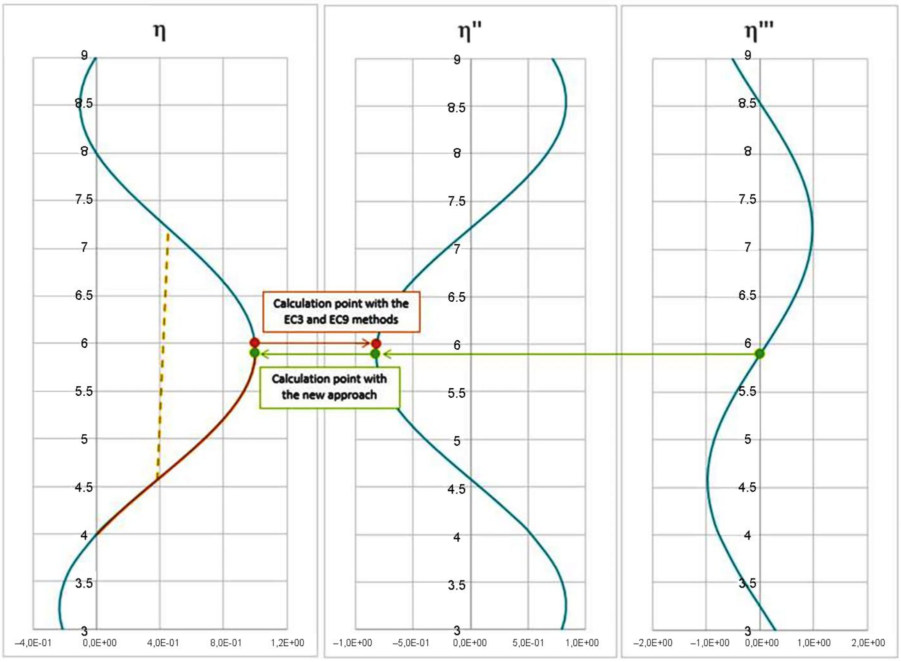


Figure 10. Calculation points of façade scaffold 8×0.7 m

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