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DOES HIGH-END HOUSING ALWAYS HAVE A PREMIUM LUXURY VALUE? A THEORETICAL AND NUMERICAL STUDY

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Abstract. Using the real options approach, we try to evaluate the luxury value inherent in high-end housing and estimate its premium returns based on the simulation of the model. The key finding of the paper is that the luxury premium from the value of the high-end housing can be identified by the real options model, which is rarely documented in the literature. In addition, the luxury value per unit size of high-end housing can be imputed through the model simulation. Based on the results, we find that the changes of the estimated value per unit size can explain the dynamic housing market behaviour in the recessions and expansions over the business cycle. The luxury value will even become negative during the recession period. In summary, the luxury premiums of high-end housing are higher than those of general housing, but not all high-end housing has positive luxury premiums. If sellers and/or builders of high-end housing cannot meet the conditions that maximize the utility of high-end housing buyers, negative returns will accrue from selling high-end housing.

Keywords: luxury value, high-end housing, luxury premium, high-end housing value, real options.

Introduction

Despite the global economic recession, the luxury goods market has been booming in the past decades. Even after the economic contraction in 2020 due to the COVID-19 pandemic, the market was expected to grow by 13% to 15% in 2021 to EUR 1.14 trillion (Bain & Company, 2021)[1]. Some wealthy consumers are motivated to consume highly conspicuous goods and services to flaunt their wealth and thereby achieve their expected social status (Veblen, 1899). Conspicuous consumption also applies to the housing market. Zahirovic-Herbert and Chatterjee (2011) found that wealthier buyers tend to pay an extra premium for the different names of the housing even though some buyers are less willing to pay during recession times. According to Leguizamon (2010), "housing lends itself very neatly to spatially determined reference groups and is also a highly visible form of consumption". As conspicuous consumption behaviors tend to cause price deviations in goods from their fundamental values (Bagwell & Bernheim, 1996), luxury houses are also likely to command higher premiums than standard residential houses. In other words, conspicuous high-end housing buyers will have to pay positive housing premiums in some cases. Lee and Mori (2016) provided empirical evidence to show that conspicuous demand has a stronger relationship with high-end housing price increases in the U.S. metropolitan statistical areas (MSAs) with a steady, higher housing premium than in MSAs with volatile and lower premiums during the boom period. Therefore, high-end housing seems capable of generating higher profits for housing builders in comparison to general residential housing.

Luxury premiums on the high-end housing price may not just come from the intrinsic value of high-end housing but can also be derived from the buyers' motivation to signal their wealth and social status. Turnbull et al. (2006) proposed that a larger house can sell at a premium when compared with otherwise identical houses in a homogenous neighborhood. Besides, luxury homes

[1] Source: Bain & Company, 2021. https://www.bain.com/insights/from-surging-recovery-to-elegant-advance-the-evolving-future-of-luxury/

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may imply higher quality of neighboring public goods and services (Downes & Zabel, 2002; Myers, 2004; Chay & Greenstone, 2005). Zahirovic-Herbert and Chatterjee (2011) analyzed the values implied by the property name like "country" and "country club" within a neighborhood. Based on the data sampled from a local housing market in U.S.A., they found that wealthier buyers tend to pay higher price premiums for a property name with "country club" than other buyers. Lee and Mori (2016) also proposed that certain types of housing gain visibility in terms of size, luxurious design, and high-quality locations with excellent neighborhood amenities. Additionally, by using the sampled data from the housing market in Israel, Levy and Snir (2018) found that, luxury housing prices are stickier and less flexible than the middle-class housing prices as the homebuyers in search of luxury will lead to price rigidities in some market segments and may affect the propagation of economic cycles.

Instead of relying on the characteristics of high-end housing such as quality, size and social-economic status of homeowners, researchers tried to examine the investment and consumption implied by the housing behavior, which are the dual motives of the general and high-end housing buyers in their decision making. Henderson and Ioannides (1983) distinguished between investment and consumption demands by maximizing housing utility with the given family budget constraints. In view of their model's assumptions and parameter setups, Bourassa (1995) and Arrondel and Lefebvre (2001) modified their model and applied it to countries such as Australia and France, with mixed results. Furthermore, Brueckner (1997) introduced a portfolio approach to analyze the optimal allocation of consumption and investment by homeowners. His approach was further expanded by Yao and Zhang (2005) to measure the effect of investment constraints on homeowners' consumption and choice of housing tenure. Their findings show that buyers and/or renters will place different weightings on liquid assets (bonds and stocks) and housing to maximize their utility from their suboptimal decision making.

As an alternative to the traditional approach in the aforementioned literature, the real options approach was applied to evaluate the option value of waiting while accounting for the uncertainty regarding the future value of investments by firms, such as investment projects under uncertainty (Bloom et al., 2007; Bloom, 2009; Dixit & Pindyck, 1994; Childs et al., 1998), land development (Titman, 1985), lease contracts (Grenadier, 1995), and the decision for homeowners to sell (Qian, 2013). For example, Qian (2013) found that the supply is constrained by the homeowners with embedded call options who will delay their trading decision in expectation of higher prices in the future. In addition, Wang et al. (2020) applied the real options to rental market to derive the supply and demand of renting houses in Hong Kong and cities in mainland China. They found that in a highly volatile housing market, the decision made by renters to buy houses and landlords to sell will affect the equilibrium rental rate through the size of shifts in supply and demand. Hung and Tzang (2021) also used the real options model to decompose housing value into consumption and investment by evaluating the put options owned by houseowners with a given set of parameters. They proposed that the comfort and utility provided by housing are critical for homeowners in deciding whether to sell their houses.

In this study, we extend the model of Hung and Tzang (2021) to derive the value of high-end housing premiums and obtain an analytical solution to the luxury value. To the best of our knowledge, our model is the first to theoretically evaluate high-end housing premiums. It also complements the deficiency in the current literature with which focuses mainly on the empirical analysis of conspicuous consumption behavior influencing high-end housing purchases. In addition, we assume that different utility rental benefits exist for high-end and general housing buyers, which helps us differentiate the premium values attributed to the housing type. Based on this assumption, we can decompose high-end housing values into consumption, investment, and luxury values, which can be derived from our proposed real-options model. Finally, based on the simulated results from the model, we find that high-end housing cannot always command higher premiums if its luxury value does not meet the demands of homebuyers.

We believe Taiwan's real estate market is ideal for lending support to our simulated results. As Taiwan is a small open economy germinated from Chinese culture, people tend to treat housing as one of the most important assets in their portfolios. Furthermore, housing in Taiwan is widely considered an investment in the family assets which can be inherited by descendants. The Taiwan Household and Population Census showed a home ownership rate of 79% in 2020. According to the Hsinyi Housing Index, 2021, housing prices have more than tripled in Taichung and Kaohsiung since 2008[2]. With the increasing popularity of high-end housing trends since 2010, many people in Taiwan have also preferred high-end housing to replace their current residential houses in order to satisfy their specific tastes for residential comfort as well as to gain possible value-addition from owning the properties. Therefore, we focus on high-end housing buyers who also decide to reside in houses so that they can maximize their utility from both owning the house and living in it[3].

[2] Taichung and Kaohsiung are the second and third largest cities in Taiwan, respectively.

[3] This is a strong assumption as some of the buyers of high-end housing may consider high-end housing just for renting or as long-term investments according to their asset al.ocation purpose. However, the number of this kind of buyers should be limited and accounts for a small group belonging to certain economic elites or people in the top ranks of the society. In this study, we focus on the larger number of housing buyers who, within their financial capacity, have the option to exchange their currently resided housing for high-end housing for residential purposes.

Section 1 presents our model and theory. Section 2 describes the simulation and numerical analysis of the model. Section 3 provides an empirical analysis of transaction data in the Taiwan luxury housing market based on the model. The last section concludes the study.

1. Model

According to Hung and So (2012), when housing buyers decide to invest in real estate, they consider prices of housing and their ability to pay for them. Essentially, there are two values provided by general housing properties: consumption and investment. Therefore, F^H, the value of general housing properties, can be separated into consumption value (HC) and investment value (F^{IG}). HC is the housing utility offered to residents in their houses. Investment value (F^{IG}) is a real option for selling housing and realizing capital gains. Whereas, F^Q, the value of high-end housing, has one additional value, which is, luxury value (HL), when compared with the general housing value (F^H). Demanders of high-end housing normally have higher requirements for houses that are different from the general ones. Richer housing buyers pay more for HL to buy high-end housing (Lee & Mori, 2016).

We used the real options model (Dixit & Pindyck, 1994) to derive the consumption, investment, and luxury values of high-end housing. As general housing value is composed of consumption value and investment value, the difference between the value of high-end housing and general housing is the luxury value. Consequently, the consumption and luxury values can be as follows:

$$HC = F^H - F^{IG}; \tag{1}$$

$$HL = F^Q - F^H. \tag{2}$$

Housing prices are also determined by housing location and region. Therefore, the average housing price index can be obtained for each region according to its population and commercialization level, and we can use this index as a proxy for housing prices[4]. The housing price index used in this study represents the mean housing price in a city. Following Dixit and Pindyck (1994), we assume that the current regional housing price index H follows a random geometric Brownian motion:

$$\frac{dH}{H} = (\mu_H - \delta_H)dt + \sigma_H dZ_t, \tag{3}$$

where μ_H is the housing price index return, δ_H is the depreciation rate of the housing price, σ_H is the return variation of the housing price index, and Z_t is the standard Brownian motion. Following Leland (1994), Dixit and Pindyck (1994), and Uhrig-Homburg (2005), we make three assumptions. First, we assume the existence of a risk-free asset with a constant rate of interest r. Second, as H is the housing price index, we assume that holding housing assets is similar to holding a housing portfolio with price proxied by H. F^i is defined as the derivative of the housing price index, which can be regarded as the portfolio cash-flow yield[5], $C_{it}dt$, where i denotes different types of housing buyers. When H is not high enough to attract owners to sell their houses, the housing portfolio continuously generates cash flow yields. Third, we exclude the explicit time dependence of H.

As F^i is the housing price index derivative,

$$0.5H^2\sigma_H^2 F_{HH}^i + (r - \delta_H)HF_H^i + F_t^i - rF^i + C_t = 0. \tag{4}$$

Equation (4) has the general solution:

$$F^i = X_0 + X_1 H^{\lambda_1} + X_2 H^{\lambda_2}, \tag{5}$$

where[6]

$$\lambda_1 = 0.5 - \frac{r-\delta_H}{\sigma_H{}^2} + \sqrt{\left(\frac{r-\delta_H}{\sigma_H{}^2} - 0.5\right)^2 + \frac{2r}{\sigma_H{}^2}} \tag{5.1}$$

and

$$\lambda_2 = 0.5 - \frac{r-\delta_H}{\sigma_H{}^2} - \sqrt{\left(\frac{r-\delta_H}{\sigma_H{}^2} - 0.5\right)^2 + \frac{2r}{\sigma_H{}^2}}. \tag{5.2}$$

X_0, X_1 and X_2 are constants determined by the boundary conditions. Any time-independent claim with F^i whose payout C_t is greater than zero, we can further examine derivative securities.

1.1. General housing value (F^H)

This study assumes that house buyers are real house demanders. We also assume that, except for the wealthiest, the high-end housing demanders are the people who will be the residents in their purchased houses. This assumption also applies to the general housing demanders. Consequently, the owner (dweller) of the house incurs implied rental income[7] and related maintenance expenses in each period. According to Equation (4), C_t can be defined as $H_0 w_H + H_0 w_H \Delta w_H - H_0 z - H\delta_H(1-t_c)$, of which the first two terms, $H_0 w_H + H_0 w_H \Delta w_H$, are proxies for periodic rent and rent variants and can also be regarded as the utility rental benefit for the general housing of house

[4] Housing price index, FAFH, is composed by Federal Housing Finance Agency every month based on the data of Freddie Mac and Fannie Mae to show the changes of housing prices in the recent two months.

[5] For housing residents, owner-occupied houses can generate rental cost savings which can be treated as the cash-flow yields. This also applies to investors' housing which can generate rental cash flow.

[6] As Equation (5) is the solution to the differential Equation (4), λ is assumed to be the root of the fundamental quadratic equation: $\mathcal{L} \equiv 0.5\sigma_H^2\lambda(\lambda-1) + (r-\delta_H)\lambda - r$, where $\lambda_1 > 1$ and $\lambda_2 < 0$ (see p. 180–181 of Dixit and Pindyck, 1994).

[7] We adopted real options model to evaluate the option value held by homeowners to decide whether to live in or sell their houses in the future. Therefore, homeowners who reside in the houses will incur implied rental income, which can be treated as an opportunity cost of living in the houses.

demanders[8]. The utility rental benefit rate of the general housing and the rental growth rate are represented by w_H and Δw_H, respectively. $H_0 z$ is the housing maintenance expense, $H\delta_H(1-t_c)$ is the after-tax depreciation expense. The house owner also must pay management costs $H_0 z$ and tax depreciation costs $H\delta_H(1-t_c)$; t_c and z are the income tax rate of the buyer and the house maintenance expense rate, respectively. The sum of the rent and future rent growth is $\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0$. We assume that housing maintenance costs are proportional to H, because maintenance costs are affected by inflation and housing market prices (Gallin, 2008; Mikhed & Zemčík, 2009; Kishor & Morley, 2015). In contrast to general housing buyers, high-end housing buyers attach more luxury value since they show off high-end housing as a luxury. Therefore, we denote the utility rental benefit as w_j, where $j = H$ or Q based on the type of housing buyer:

1. Rental expenses: If house demanders do not own any housing, they would live in a rental house and incur rental expenses. Consequently, w_j is equal to w_H which is the expense rate that homeowners can save by buying their own houses instead of renting houses. This rental expense (or rental revenue) applies to both general and high-end housing buyers.
2. Satisfaction with high-end housing buyers: There are many types of high-end housing in the housing market, each of which has unique features that are attractive to high-end housing buyers. Therefore, they are more concerned about the satisfaction derived from conspicuous housing expenditure. Satisfaction, that is, the utility rental benefit, can be represented as w_Q.

In addition, we also assume w_Q to be dependent on the economic condition. Homebuyers are more willing to pay more to gain higher satisfaction (higher w_Q) from the luxury investment on housing in the booming period than they are in the recession period. This is also consistent with the rationality of consumption behaviour sensitive to the economic cycle. In other words, during the economic recession, the price of housing with high luxury investment may show some rigidity that makes it undervalued by homebuyers (Levy & Snir, 2018).

For general housing buyers, w_H is the utility of general housing rental costs which does not satisfy the utility condition of conspicuous consumption. However, for high-end buyers, w_Q is the utility rental benefit for high-end housing buyers and w_Q includes more conspicuous utility rental benefits than w_H. By not complicating the quantitative analysis of utility costs, we assume that w_j is an exogenous variable[9] applied to the utility rental benefit rate for housing.

Let H_L denote the housing price level at which housing owners are willing to sell and realize their capital gains. According to the real options model, when H approaches targeted H_L, a homeowner sells their houses and receives after-tax capital gains, as shown by the boundary condition in Equation (6.1). However, as H decreases, homeowners would continue to live in their homes. Condition (6.2) holds that the decision to sell becomes irrelevant as H decreases, and the value of general housing approaches the value of the net rental house. The values and boundary conditions of F^H are as follows:

$$H = H_L,\ F^H = H_L - (H_L - H_0)\times t_c; \tag{6.1}$$

$$H \to 0,\ F^H = \left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0 - H(1-t_c). \tag{6.2}$$

According to Equation (5), it is apparent that X_2 should be equal to 0 when H approaches zero with negative λ_2. As $H^{\lambda_1}\to 0$ as $H \to 0$, together with Equation (6.2), this implies that $X_0 = \left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0 - H(1-t_c)$. Finally, using Equation (6.1), $X_2 = \left[H_L - (H_L - H_0)\times t_c\right] - \left[\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0 - H\times(1-t_c)\right]\times\left(\frac{1}{H_L}\right)^{\lambda_1}$. In this case, Equation (5) and the boundary conditions of the subjective value of the general house price F^H are as follows:

$$F^H = \left[\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0 - H\times(1-t_c)\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right] + \left[H_L - (H_L - H_0)\times t_c\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{7}$$

Equation (7) can also be written as $F^H = \left[\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0 - H\times(1-t_c)\right]\times(1-P_L^H) + \left[H_L - (H_L - H_0)\times t_c\right]\times P_L^H$, where $P_L^H \equiv \left(\frac{H}{H_L}\right)^{\lambda_1}$ represents the probability that high-end housing buyers sell their houses at H_L.

Interest tax shield and mortgage value: F^{int} and F^D

Most homebuyers use mortgage loans to buy houses, which generate interest expenses and tax shields. Therefore, we can calculate the sum of the present value of interest payments in the entire loan period as the difference between the sum of the present value of the mortgage payments in each period and the original loan balance, as represented in Equation (8.1). The mortgage payment PMT given by the standard annuity formula is represented in Equation (8.2) as follows:

$$\left(\int_0^T PMT\times e^{-rt}dt - M_0\right)\times t_c; \tag{8.1}$$

$$PMT = \frac{M_0\times y}{1-e^{-yT}} = \frac{H_0\times(1-d)\times y}{1-e^{-yT}}, \tag{8.2}$$

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[8] Here w_H is the same as w denoted in Hung and Tzang (2021).

[9] The rental utility can be estimated according to Campbell and Cocco (2015)'s model. As it is not the main concern of this study, we assume that w_j is an exogenously set variable.

where d is the required down payment, y is the interest rate the household pays on a fixed-rate mortgage with maturity T, M_0 is the original loan balance, t_c is the income tax rate of the buyer.

When H increases to H_L, homeowners would sell their house and prepay the mortgage so that they do not have to pay any interest on the mortgage. When H decreases, the homeowner will continue to pay the interest and mortgage principal. Unlike investments in the security market, which may induce a stop-loss strategy, homeowners will not sell their houses even though the housing prices take a hit because we assume in the model that homeowners are also residents of the house. F^{int} is defined as the tax-sheltering value of the interest payments carried by mortgage loans. According to Equation (4), C represents all payments of mortgage-deducted principal $\left(\int_0^T PMT \times e^{-rt} dt - M_0\right) \times t_c$ and the boundary conditions are as follows:

$$H = H_L,\ F^{int} = 0; \tag{9.1}$$

$$H \to 0,\ F^{int} = \left(\int_0^T PMT \times e^{-rt} dt - M_0\right) \times t_c. \tag{9.2}$$

Equation (9.1) reflects the loss of tax-shelter benefits if the homeowner sells the home. Using Equation (5), the boundary conditions above can be reformulated as

$$F^{int} = \left(\frac{PMT}{r}\left(1 - e^{-rT}\right) - M_0\right) \times t_c \times \left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right]. \tag{10}$$

F^{int} is a decreasing, strictly convex function of H. F^D is defined as the mortgage value, which is, the payments of the mortgage. According to Equation (4), where C_t is $\int_0^T PMT \times e^{-rt} dt$ the boundary conditions are as follows:

$$H = H_L,\ F^D = M_0; \tag{11.1}$$

$$H \to 0,\ F^D = \int_0^T PMT \times e^{-rt} dt. \tag{11.2}$$

By the boundary conditions, Equation (5) has a solution:

$$F^D = \frac{PMT}{r}\left(1 - e^{-rT}\right) \times \left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right] + M_0 \times \left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{12}$$

Transfer costs F^k

We assume that when high-end house owners want to sell in their houses and move to another house, they incur a transfer cost F^k which is assumed to be a proportion β of the house selling price H_L. According to Equation (4), where C_t is 0, the boundary conditions apply:

$$H = H_L,\ F^k = \beta H_L; \tag{13.1}$$

$$H \to 0,\ F^k = 0. \tag{13.2}$$

In this case, Equation (4) and boundary conditions have a solution:

$$F^k = \beta H_L \left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{14}$$

The transfer cost is an increasing, strictly concave function of H. Equation (14) can also be represented as $F^k = \beta H_L P_L^H$, implying that the current value of the transfer cost is proportional to the selling price multiplied by the selling probability.

Optimal decision for general housing owners

The general housing owner chooses a level of H to maximize the current value of home equity. According to Equation (15), the home equity for general house owners is the housing value plus interest tax shield value less transfer costs and mortgage value:

$$E = F^H + F^{int} - F^k - F^D = \left[\begin{array}{l}\left(\frac{w_H}{r - \Delta w_H} - \frac{z}{r}\right)H_0 - H \times \left(1 - t_c\right) + \\ \left(\frac{PMT}{r}\left(1 - e^{-rT}\right) - M_0\right) \times t_c - \frac{PMT}{r}\left(1 - e^{-rT}\right)\end{array}\right] \times \left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right] + \left[H_L - \left(H_L - H_0\right) \times t_c - M_0 - \beta H_L\right] \times \left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{15}$$

when H approaches H_L, general housing owners are more likely to sell their houses and earn a profit. Thus, the higher possible value H_L for consistent, positive home equity value for all $H < H_L$ is such that $\partial E / \partial H_L |_{H=H_L} = 0$: a "smooth-pasting" condition (Leland, 1994) at $H = H_L$. Differentiating Equation (15) with respect to H_L and setting the expression equal to zero with $H = H_L$, we can solve for the optimal selling price for $H_L^{\#}$:

$$H_L^{\#} = \frac{\left(\frac{w_H}{r - \Delta w_H} - \frac{z}{r}\right)H_0 \lambda_1 - \left(\frac{PMT}{r}\left(1 - e^{-rT}\right) - M_0\right) \times \left(1 - t_c\right)\lambda_1 - \lambda_1 H_0 \times t_c}{2 \times \left(1 - t_c\right)\lambda_1 - \beta\lambda_1 - \left(1 - t_c - \beta\right)}. \tag{16}$$

Proof: See Appendix A.

According to Equations (16) and (7), we can solve for the optimal ordinary housing value:

$$F^H = \left[\left(\frac{\left(w_H - z\right)}{r} + \frac{w_H \Delta w_H}{r - \Delta w_H}\right)H_0 - H_L^{\#} \times \left(1 - t_c\right)\right] \times \left[1 - \left(\frac{H}{H_L^{\#}}\right)^{\lambda_1}\right] + \left[H_L^{\#} - \left(H_L^{\#} - H_0\right) \times t_c\right] \times \left(\frac{H}{H_L^{\#}}\right)^{\lambda_1}. \tag{17}$$

Equation (17) can also be written as

$$F^H = \left[\left(\frac{w_H}{r - \Delta w_H} - \frac{z}{r}\right)H_0 - H_L^{\#} \times \left(1 - t_c\right)\right] \times \left(1 - P_L^H\right) + \left[H_L^{\#} - \left(H_L^{\#} - H_0\right) \times t_c\right] \times P_L^H,$$

where $P_L^H \equiv \left(\frac{H}{H_L^{\#}}\right)^{\lambda_1}$ represents the probability that general housing owners would sell their houses as H increases.

1.2. High-end housing value (F^Q)

As we assume that high-end housing buyers are also residents of the house, utility rental benefit w_Q can be measured in our model. In general, the utility rental benefit w_Q can be affected by many features of the houses, such as location, luxury status of the building, structural safety, high security, luxury private facilities, and luxury public facilities for the building. Because high-end housing buyers are willing to pay a higher price for high-end housing than for general housing, builders invest αH_0 in high-end housing than in general housing. When a builder has constructed featured and stylish high-end housing that is different from general housing, the utility rental benefit w_Q could be higher if high-end housing buyers favor the builder's high-end housing.

According to Equation (4), $H_0 w_Q + H_0 w_Q \Delta w_Q$ refers to the utility rental benefit of high-end housing for buyers. The utility rental benefit rate of high-end housing and its growth rate are denoted as w_Q and Δw_Q, respectively. The sum of the rent and future rent change present value can be represented as $\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0$. We assume that maintenance costs are relative to H because they are affected by inflation and housing market prices. Furthermore, the homeowner must pay management costs $H_0 z$ and tax depreciation costs or maintenance costs $H\delta_H(1+\alpha)(1-t_c)$, where t_c and z are the income tax rate of the buyer and the maintenance expense rate of the house, respectively.

When H increases to the targeted $\bar{H}_L$, homeowners would sell their houses and realize capital gains, but when H decreases, homeowners would continue to reside in the houses, and the value and boundary conditions of F^Q are as follows:

$$H=\bar{H}_L,\ F^Q=\bar{H}_L\times(1+\alpha)-\left(\bar{H}_L(1+\alpha)-H_0(1+\alpha)\right)t_c; \tag{18.1}$$

$$H\to 0,\ F^Q=\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c). \tag{18.2}$$

In this case, Equation (5) and the boundary conditions of the subjective value of the high-end house price F^Q are given as follows:

$$F^Q=\left[\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)\right]\times\left[1-\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}\right]+$$
$$\left[\bar{H}_L(1+\alpha)-\left(\bar{H}_L(1+\alpha)-H_0(1+\alpha)\right)t_c\right]\times\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}. \tag{19}$$

Conspicuous consumption value (F^{UR})

As we assume that high-end housing buyers are conspicuous consumers, their demands differ from those of general housing buyers. High-end housing buyers prefer to flaunt their wealth (Lee & Mori, 2016). We assumed that high-end housing owners would require an extra conspicuous consumption value, F^{UR}, included in their selling threshold. This selling threshold also guarantees that high-end housing owners achieve the minimum required rate of return R based on the selling threshold. Because high-end housing owners have a special taste and preference for high-end housing, they are very likely to hold and reside in their houses unless they can realize the minimum required return R for their high-end housing.

When high-end housing demanders purchase high-end houses, they often invest large expenses in furnishing and decorating. We assume housing prices have a γ ratio. When H approaches $\bar{H}_L$, homeowners are more likely to sell their houses and realize capital gains. When H decreases, however, homeowners would continue to reside in their houses, and the boundary conditions of F^{UR} are as follows:

$$H=\bar{H}_L,\ F^{UR}=H_0\times(1+\gamma)\times R; \tag{20.1}$$

$$H\to 0,\ F^{UR}=0. \tag{20.2}$$

Using Equation (4), where C_t is zero, the boundary conditions above yield the following:

$$F^{UR}=\left[H_0\times(1+\gamma)\times R\right]\times\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}. \tag{21}$$

High-end housing optimum decision

To maximize the current value of home equity E, high-end housing owners sell their houses at price $\bar{H}_L$. The home equity value of high-end housing is defined as the high-end housing value plus the interest tax shield value less the transfer costs and mortgage value ($E=F^Q+F^{int}-F^k-F^D$). High-end housing owners decide to sell their houses at $\bar{H}_L$ which is different from the asking price H_L of general housing owners. The key difference was F^{UR}. As mentioned above, we assume that the utility from home equity is $E^u = E - F^{UR}$, and E^u can be derived as follows:

$$E^u=F^Q+F^{int}-F^k-F^D-F^{UR}=$$
$$\left[\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)\right]\times$$
$$\left[1-\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}\right]+\left[\bar{H}_L(1+\alpha)-\left(\bar{H}_L(1+\alpha)-H_0(1+\alpha)\right)t_c\right]\times$$
$$\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}+\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c\times\left[1-\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}\right]-\frac{PMT}{r}\left(1-e^{-rT}\right)\times$$
$$\left[1-\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}\right]-M_0\times\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}-\beta H_L\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}-\left[H_0\times(1+\gamma)\times R\right]\times\left(\frac{H}{\bar{H}_L}\right)^{\lambda_1}, \tag{22}$$

when H increases to $\bar{H}_L$, high-end housing buyers may consider selling the house to realize capital gains. Thus, the higher possible value $\bar{H}_L$ for consistent and positive home equity values of the high-end housing owners for all $H<\bar{H}_L$ are such that $\partial E^u/\partial H\big|_{H=\bar{H}_L}=0$, a "smooth-pasting" condition (Leland, 1994) at $H = \bar{H}_L$. Differentiating Equation (22) with respect to $\bar{H}_L$, we set the expression equal to zero with $H = \bar{H}_L$ to solve for $\bar{H}_L$:

$$\left.\frac{\partial E^u}{\partial H}\right|_{H=\bar{H}_L} = \left.\frac{\partial\left(F^H + F^{int} - F^k - F^D - F^{UR}\right)}{\partial H}\right|_{H=\bar{H}_L} = 0\,. \quad (23)$$

We can solve for the optimal selling house price H_L^* that maximizes the home equity value of high-end housing buyers as follows:

$$H_L^* = \frac{\left[\left(\frac{w_Q}{r-\Delta w_Q} - \frac{z}{r}\right)H_0 - \left(\frac{PMT}{r}\left(1-e^{-rT}\right) - M_0\right)\left(1-t_c\right) - H_0\left(1+\alpha\right)t_c + H_0 \times \left(1+\gamma\right) \times R\right]\lambda_1}{\left(1+\alpha\right)\left(1-t_c\right)\lambda_1 + \left[\left(1+\alpha\right)\left(1-t_c\right) - \beta\right]\left(\lambda_1 - 1\right)}. \quad (24)$$

Proof: See Appendix B.

Using Equation (24), H_L^* is affected by factors such as w, t_c, r, z, δ_H, σ_H, H_0, T, PMT and M_0. We also note that the regional housing price index, H_L^*, at which selling occurs,

1. increases with σ_H, w_Q, Δw_Q, H_0, y, t_c, β, γ, R and M_0;
2. decreases with z, r, T and δ_H.

According to Equations (19) and (24), we can solve for the optimal high-end housing value:

$$F^Q = \left[\left(\frac{w_Q}{r-\Delta w_Q} - \frac{z}{r}\right)H_0 - H\left(1+\alpha\right)\left(1-t_c\right)\right] \times \left[1 - \left(\frac{H}{H_L^*}\right)^{\lambda_1}\right] + \left[H_L^*\left(1+\alpha\right) - \left(H_L^*\left(1+\alpha\right) - H_0\left(1+\alpha\right)\right)t_c\right] \times \left(\frac{H}{H_L^*}\right)^{\lambda_1}. \quad (25)$$

Equation (25) can also be written as

$$F^Q = \left[\left(\frac{w_Q}{r-\Delta w_Q} - \frac{z}{r}\right)H_0 - H\left(1+\alpha\right)\left(1-t_c\right)\right] \times \left(1 - P_L^Q\right) + \left[H_L^*\left(1+\alpha\right) - \left(H_L^*\left(1+\alpha\right) - H_0\left(1+\alpha\right)\right)t_c\right] \times P_L^Q,$$

where $P_L^Q \equiv \left(\frac{H}{H_L{}^*}\right)^{\lambda_1}$ represents the probability that high-end housing owners would sell their houses as H approaches $\bar{H}_L$.

2. Numerical analysis

In this section, we provide simulated results to ensure the consistency of the luxury behavior of high-end housing buyers that is commonly observed in the real world. Table 1 shows the initial values of all parameters assumed for the baseline case. Furthermore, by referencing Campbell and Cocco (2015), we adjusted the values of some of the parameters according to Taiwan's real estate market condition to measure the degree to which the simulated results of the models are affected by the parameter values in the model. Part of the initial values of parameters are based on the regulations of Ministry of the Interior of Taiwan (MIT) and the report of ROC Real Estate Appraisers Association (ROCREAA)[10].

In Panel A of Table 1, we assume the housing price depreciation rate to be 0.02 per year because the service life of Taiwan's housing is estimated to be 50 years[11]. The house price index return variation (0.162) is computed using historical data from the Taiwan housing price index. For calculation, the housing price index is initialized at 100. In Panel B, the utility rental benefit rate of general housing (w_H) is assumed to be 2.5% of the housing price, which represents the widely accepted annual rental yield in the Taiwanese housing market. However, we assumed a slightly higher rate of 3.5% for the utility rental benefit of high-end housing (w_Q). In Panel C, the original loan balance (M_0) is assumed to be 70, as we initialize the housing price to 100 (H_0). Relocation fee (β) also includes refurnishing fees. In proportion to the housing price, α is the additional investment by builders in high-end housing to satisfy buyers' conspicuous consumption demands.

By Equation (2), luxury values differ between high-end and general housing prices. We grouped the values in Table 2 into four regions with gray shading on the table corners, defined as A, B, C, and D, as shown in Table 3, to summarize the subjective values of high-end housing in different levels of α and w_Q. Lee and Mori (2016) mentioned that when luxury values are high, a builder gains more benefits. They also proposed that conspicuous consumption behavior has a much more significant, positive relationship with high-end housing premiums among MSAs, including the top 30% of the high-end housing premiums, when compared to MSAs, including the bottom 30% high-end housing premium group. As a result, not all high-end housing has the same premium. However, as Lee and Mori did not clearly distinguish the values among different types of high-end housing, we analyzed the values of four types of high-end housing for further analysis.

Table 2 summarizes the changes in the subjective values of high-end housing with α according to the various levels of buyers' utility rental benefit w_Q. We find that for a given w_Q, a higher α will lead to lower subjective values of high-end housing. In other words, the more the builder invests in the construction of the housing, the more high-end housing buyers would have to pay for that "particular"

[10] ROCREAA is a non-profit organization to provide fair and trustworthy land and real estate valuation information in the Taiwan real estate market. ROCREAA publishes monthly economic report and rules of evaluation based on the most current economic data. Technical issues in real estate evaluation like service tenure of building, residual values, land development fees, cost of management and sales are also articulated in its report for appraisers' reference.

[11] Directorate General of Budget, Accounting and Statistics, Executive Yuan of Taiwan decreed that the service life of Taiwan residential houses is 55 years. We use 50 years for simulation purposes only.

Table 1. Baseline parameters*

Description	Parameter	Value
Panel A: Housing price		
Housing price depreciation rate	δ_H	0.02
Housing price index return variation	σ_H	0.162
Initial housing price index	H_0	100**
Panel B: High-end housing buyer utility value		
Utility rental benefit rate of general housing	w_H	0.025
Utility rental benefit rate of general housing growth rate	Δw_H	0
Utility rental benefit rate of high-end housing	w_Q	0.035
Utility rental benefit of high-end housing growth rate	Δw_Q	0
Panel C: Housing expense and tax		
Income tax rate of buyer	t_c	0.2
Maintain expense rate	z	0.005
Rate of interest	r	0.01
Required down payment	d	0.3
Interest rate that household pays on the fixed-rate mortgage	y	0.02
Maturity of mortgage (in year)	T	20
Original loan balance	M_0	70
Relocation fee rate	β	0.1
Additional investment on housing (proportion of H_0)	α	0.2
Luxuriously furnished expense ratio	γ	0.3

Note: *Part of the initial values of parameters are based on the regulations of Ministry of the Interior of Taiwan (MIT) and the report of ROC Real Estate Appraisers Association (ROCREAA). ROCREAA is a non-profit organization to provide fair and trustworthy land and real estate valuation information in the Taiwan real estate market. ROCREAA publishes monthly economic report and rules of evaluation based on the most current economic data such as business indicators, interest rate, Taiwan Manufacturing PMI, Consumer Confidence Index, etc. Technical issues in real estate evaluation like service tenure of building, residual values, land development fees, cost of management and sales are also articulated in its monthly report for appraisers' reference. **For comparison purpose, we set the housing price index to 100.

Table 2. Value of high-end housing*

	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7
w_Q = 2.5%	112.05	104.06	96.07	88.09	80.10	72.12	64.14
w_Q = 3.0%	162.05	154.06	146.07	138.09	130.10	122.12	114.14
w_Q = 3.5%	212.05	204.06	196.07	188.08	180.10	172.12	164.14
w_Q = 4.0%	262.05	254.06	246.07	238.08	230.10	222.12	214.14
w_Q = 4.5%	312.05	304.06	296.07	288.08	280.10	272.12	264.14
w_Q = 5.0%	362.05	354.06	346.07	338.08	330.10	322.12	314.14
w_Q = 5.5%	412.04	404.06	396.07	388.08	380.10	372.12	364.13
w_Q = 6.0%	462.04	454.06	446.07	438.08	430.10	422.11	414.13

Note: *This table reports how the subjective values of high-end housing will change with α according to different levels of buyers' utility rental benefit w_Q. α is the builder's additional investment in proportion to H_0.

housing, which may not fully satisfy the utility of high-end housing buyers and thus cause a fall in their subjective values for high-end housing. If the builder's additional investment can also increase the utility of the rental benefit of high-end housing buyers (w_Q), the subjective value of such high-end housing will rise. For example, when α = 0.1 and w_Q = 2.5%, the subjective value of high-end housing is 112.05. When α increased to 0.2, and w_Q increased to 3.0%, the subjective value of high-end housing rises to 154.06.

Alternately, even a minimal investment in high-end housing (α = 0.1) can deliver higher subjective values of the housing to buyers (from 112.05 to 462.04), when the investment not only satisfies the utility rental benefit of the high-end housing buyers but also raises their benefit level (from 2.5% to 6%). More interesting observations can be made from the boom-bust cycle of the economy. For example, top-left block in Table 2, which can be regarded as the economic recession period, shows that developers tend to make less investment in high luxury

Table 3. Different types of high-end housing*

	Small α (small investment)	Large α (large investment)
Low w_Q	Type D: Lowest level	Type B: Not worthy of the name
High w_Q	Type C: Quality level	Type A: Highest level

Note: *α is the builder's additional investment in proportion to H_0. w_Q is the buyers' utility rental benefit.

Table 4. Luxury value of high-end housing*

	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7
w_Q = 2.5%	6.44	−1.55	−9.53	−17.52	−25.50	−33.48	−41.46
w_Q = 3.0%	56.44	48.45	40.47	32.48	24.50	16.52	8.54
w_Q = 3.5%	106.44	98.45	90.47	82.48	74.50	66.52	58.53
w_Q = 4.0%	156.44	148.45	140.47	132.48	124.50	116.51	108.53
w_Q = 4.5%	206.44	198.45	190.47	182.48	174.50	166.51	158.53
w_Q = 5.0%	256.44	248.45	240.47	232.48	224.50	216.51	208.53
w_Q = 5.5%	306.44	298.45	290.47	282.48	274.49	266.51	258.53
w_Q = 6.0%	356.44	348.45	340.46	332.48	324.49	316.51	308.53

Note: *This table reports how the luxury values of high-end housing will change with α according to different levels of buyers' utility rental benefit w_Q. α is the builder's additional investment in proportion to H_0.

housing. Meanwhile, homebuyers with low utility rental benefit will be more easily satisfied with minimal investment in luxury housing. When the economy is gradually recovering, homebuyers will move into bottom-left block in Table 2. They will demand higher utility rental benefit derived from luxury housing, thus raising the housing value in the market. In this period, however, developers are not able to increase the supply in time for homebuyers of high-end housing and, therefore, homebuyers will only have to consume the inventories of luxury housing accumulated in the recession period.

According to Table 3, Types A and B are the housing with more luxury investments, whereas Types C and D are the housing with less luxury investments[12]. From the perspectives of high-end housing buyers in the boom-bust economic cycle, they will demand higher w_Q from Types A and C than from Types of B and D. Housing luxury values and luxury-premium returns can also be computed for these four types of housing (see Tables 4 and 5). High-end housing of Type A is housing in which builders make the highest amount of investment and can fully satisfy the buyer's utility rental benefit. Type B is not worthy of the name high-end housing because the builder's investment seems incapable of receiving appreciation from high-end housing buyers. In other words, they did not like this type of housing. Type C is defined as the quality level, which has attractive features such as location, pleasant environment, and/or transportation convenience to meet the demands of high-end housing buyers, and the builders do not have to invest a lot in the construction of the building (Kiel & Zabel, 2008). Type D has the lowest level; the builder's investment is low, and the high-end housing buyer can accept the quality of this building but will not pay much money to buy the housing.

To realize the dream of owning a luxury home, high-end housing buyers allocate an excess amount of wealth to their high-end housing. They will pay higher premiums to buy houses that satisfy their utility from conspicuous housing. Type A high-end housing can create more luxury value, and buyers would be more willing to pay higher prices to buy such high-end housing, especially in the economic booming period. However, Table 4 shows that Type C high-end housing can create the highest luxury value compared with Type A. We propose that because in Type C, high-end housing buyers will gain higher utility rental benefit with from small α when the economy condition is recovering from the recession. By contrast, Types B and D can only create low or negative luxury values as the economy is reversing from top or trapped in a deep recession. In summary, not all high-end housing can create high and positive luxury values.

Although Type D can deliver a positive luxury value when investment α is minimal (α = 0.1), the builder may also receive negative premium returns on that investment in high-end housing (−35.57% when w_Q = 2.5% in Table 5). When α rises to 0.7, the premium return (loss) can be as low as −159.23%. Therefore, it is important that builders analyze high-end housing buyers' preferences. If investment in high-end housing can evoke interest, the builder can increase the value of w_Q for high-end housing buyers.

According to Table 5, Type C high-end housing creates the highest premium returns. When w_Q = 6.0% and

[12] As we have shown in Table 2, types A, B, C and D correspond to the economic boom-bust cycle. Types A and B, located in the bottom- and top-right blocks in Table 2, can be regarded as the economic boom period. However, type A is in the economic boom period whereas type B is in the economic reversal period. Type D is in the economic recession. Type C is in the economy recovering from the recession.

Table 5. Luxury premium returns of high-end housing based on the initialized parameter values in Table 1*

	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7
w_Q = 2.5%	–35.57%	–107.73%	–131.77%	–143.79%	–151.00%	–155.81%	–159.23%
w_Q = 3.0%	464.43%	142.27%	34.89%	–18.80%	–51.00%	–72.47%	–87.81%
w_Q = 3.5%	964.43%	392.27%	201.56%	106.20%	48.99%	10.86%	–16.38%
w_Q = 4.0%	1464.43%	642.27%	368.22%	231.20%	148.99%	94.19%	55.05%
w_Q = 4.5%	1964.42%	892.27%	534.89%	356.20%	248.99%	177.52%	126.48%
w_Q = 5.0%	2464.42%	1142.27%	701.55%	481.20%	348.99%	260.85%	197.90%
w_Q = 5.5%	2964.42%	1392.26%	868.22%	606.20%	448.99%	344.19%	269.33%
w_Q = 6.0%	3464.41%	1642.26%	1034.88%	731.20%	548.99%	427.52%	340.76%

Note: *The luxury premium of high-end housing is $\frac{H_L^* - (1+\alpha)H_0}{(1+\alpha)H_0}$.

Table 6. Subjective value difference between high-end housing and general housing*

	w_H = 2.5%	w_H = 3.0%	w_H = 3.5%	w_H = 4.0%	w_H = 4.5%	w_H = 5.0%	w_H = 5.5%	w_H = 6.0%
w_Q = 2.5%	–1.55	–12.69	–26.24	–41.02	–56.50	–72.42	–88.61	–105.01
w_Q = 3.0%	48.45	37.31	23.76	8.98	–6.50	–22.42	–38.61	–55.01
w_Q = 3.5%	98.45	87.31	73.76	58.98	43.50	27.58	11.39	–5.01
w_Q = 4.0%	148.45	137.31	123.76	108.98	93.50	77.58	61.38	44.99
w_Q = 4.5%	198.45	187.31	173.76	158.98	143.50	127.58	111.38	94.99
w_Q = 5.0%	248.45	237.31	223.76	208.98	193.50	177.58	161.38	144.99
w_Q = 5.5%	298.45	287.31	273.76	258.98	243.50	227.58	211.38	194.99
w_Q = 6.0%	348.45	337.31	323.76	308.98	293.50	277.58	261.38	244.99

Note: *This table displays the subjective value difference between high-end (F^Q) and general housing (F^H) based on utility rental benefit w_Q and w_H.

α = 0.1, the premium return is approximately 3464.41%. Even with a low α, Type C has the highest value w_Q because of some specific features, such as excellent location (Kiel & Zabel, 2008). Type A can also create higher premium returns but is not as high as that of Type C. When w_Q = 6.0% and α = 0.7, the premium return for Type A is approximately 340.76%. Type D can improve its premium return from negative to positive when w_Q increases from 2.5% to 3.5%, and α increases from 0.1 0.3. The premium returns generated within this region can be improved from the lowest level of –131.77% (α = 0.3, w_Q = 2.5%) to 964.43% (α = 0.1, w_Q = 3.5%). In view of Type D high-end housing, which has the lowest values of w_Q and α, builders should focus on how to increase w_Q with a slight increase in investment to obtain higher rewards.

Table 6 lists the values of F^Q – F^H. Generally, F^Q is higher than F^H. However, when w_Q and w_H are very low, F^Q is less than F^H. It is possible that high-end housing buyers do not pay more to buy high-end housing with low w_Q. When w_H remains unchanged, the price difference increases by 50 for every 0.5% increase in w_Q. When w_Q is unchanged, the price difference decreases by approximately 15 for every 0.5% increase in w_H. According to the analysis above, the marginal contribution of the utility of high-end housing buyers is higher than that of ordinary housing buyers.

Figure 1 summarizes these findings. The subjective values of luxury premium returns for high-end housing increase with w_Q but decrease with α. For luxury premium returns on high-end housing, w_Q is a crucial factor. Before builders decide to build high-end housing, they must investigate the preferences and demands of buyers. When builders want to invest in high-end housing, they can increase w_Q by α. If they cannot fully understand high-end housing buyers' demand for high-end housing, they could suffer from investment losses, and the cost of the losses is much higher than the value of the house.

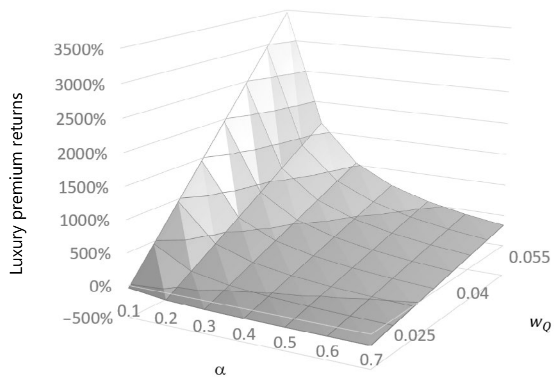

Figure 1. The changes of luxury premium returns across utility rental benefit w_Q and investment on high-end housing α

3. Empirical results

Table 7 is the summary statistics of the housing transaction data from 2016/01 to 2018/08 in the three biggest cities in Taiwan: Taipei, Taichung, and Kaohsiung, of which the number of the population is around 2.7 million equally for each city as of 2022. The datasets are collected from the Ministry of the Interior of Taiwan (MIT) and the ROCREAA. The transaction data is limited to residential homes in high-rise buildings sold in contiguous regions within the urban elite area. This study uses a sample consisting of broker-assisted high-end housing transactions within this sampled period.

We divide the high-end housing transaction data into high- and middle-to-low floors[13]. High-end housing buyers prefer high-floor homes to middle-to-low-floor ones[14], but the construction costs for high floors are higher than those for middle-to-low floors. We assume that high-end housing demanders have a higher utility rental benefit w_Q for high-floor housing owing to better view and light. Table 7 presents a summary of the sample data. The mean value is the average trading price per square meter (m^2) in New Taiwan Dollar (NTD) for Taipei, Taichung and Kaohsiung, and the prices of higher floors are higher than middle-to-low floors. We noted that the mean price of high floors in Taipei is NTD 401,127, which is significantly higher than the building cost (NTD 244,511). The price and cost spread in Taipei is also the highest (156,617 and 114,697, respectively). The cost of building includes land, construction, management, and sales costs, and we appraise it according to ROCREAA rules.

Table 7 also shows that the cost of building high floors is greater than the cost of building middle-to-low floors; however, there is a slight difference in Taichung city where the cost of buildings with middle-to-low floors is higher than that of buildings with high floors. We think that this is due to the buyers' preference for middle-to-low floors, as Taichung City experienced a severe earthquake on September 21, 1999. Another reason is the limited number of transaction data (36 and 27) from Taichung city when compared with the other two cities (126 and 75 in Taipei; 609 and 528 in Kaohsiung). During the sampling period, many high-rise buildings were still under construction with luxury housing units in the central areas of Taichung such as the 7th land readjustment zone. The land and construction costs of high floors are much higher than the costs of middle-to-low floors. Therefore, the sampled data of the construction cost in Taichung may not be representative of the Taichung market.

Table 8 lists the luxury premium returns of high-end housing of Taipei, Taichung, and Kaohsiung. By Table 8, the luxury premium returns of the high floors of Taipei and Taichung are higher than those of the middle-to-low floors, and luxury premium returns decrease as α increases. This finding is consistent with the results presented in Table 5. When $\alpha = 0.6$, luxury premium returns for high-end housing for both Taichung and Kaohsiung are negative. In Kaohsiung, the housing prices for high floors were higher than those of middle to low floors. However, the fact that the construction cost of high floors is higher than that of middle-to-low floors does not warrant a proportional increase in the luxury premium returns of high

Table 7. Summary statistics of the housing transaction data from Jan 2016 to August 2018 in three biggest cities in Taiwan

City	Description	Quantity	Cost of building (NTD)		Price/m^2 (NTD)		Diff. in means (p-value)
			Mean	Std. dev.	Mean	Std. dev.	
Taipei	High floors	126	244,511	20,761	401,127	59,693	156,617*** (7.9E-21)
	Middle-to-low floors	75	209,379	36,456	324,076	88,476	114,697*** (2.0E-06)
Taichung	High floors	36	130,373	20,096	188,159	28,217	57,786*** (0.001211)
	Middle-to-low floors	27	140,280	17,625	145,925	26,021	5,646 (0.70084)
Kaohsiung	High floors	609	53,600	4,444	82,586	15,975	28,986*** (3.124E-67)
	Middle-to-low floors	528	48,728	5,189	82,287	23,492	33,559*** (2.179E-43)

Note: ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

[13] In Taiwan, low floors of the housing are the floor from the 1st to the 5th and middle floors from the 6th to the 10th.

[14] In Taiwan, housing on higher floors symbolizes the level of luxury for housing. Housing price increases as the height of the floor increases. However, high floors also suffer from the risks of fire escape and earthquakes in Taiwan.

Table 8. Luxury premium returns of high-end housing of Taipei, Taichung, and Kaohsiung

	α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6
Taipei							
Middle-to- low floors	60.22%***	45.66%***	33.52%***	23.25%***	14.44%**	6.81%	0.14%
High floors	61.86%***	47.15%***	34.88%***	24.51%***	15.62%***	7.91%**	1.16%
Taichung							
Middle-to-low floors	–29.12%***	–29.94%***	–30.62%***	–31.19%***	–31.68%***	–32.11%***	–32.49%***
High floors	49.63%***	36.02%***	24.69%***	15.1%**	6.88%***	–0.25%**	–6.48%
Kaohsiung							
Middle-to- low floors	59.22%***	44.74%***	32.68%***	22.48%***	13.73%***	6.15%***	-0.49%
High floors	54.61%***	40.56%***	28.84%***	18.93%**	10.44%***	3.07%***	–3.37%***

Note: ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively. This table displays the subjective values of high-end housing classified by two types of housing, and α is the builder's initial investment compared to the proportion of H_0. The luxury premium of high-end housing computed is $\left(H_L^* - (1+\alpha)H_0\right) / \left((1+\alpha)H_0\right)$.

floors in Kaohsiung. This may be due to the high-end housing buyers in Kaohsiung, who are not willing to pay more for the luxury premium. In addition to the measurement of high-end housing buyers' utility rental benefit w_Q, we believe that builders may also have to consider possible factors affecting residents' preferences, such as income, spatial location, and scarcity of green environments across different cities.

Since the average income in Taipei and Taichung is higher than that in other cities, residents are willing to pay for the luxury premium. The most important decision of high-floor housing suppliers is to measure high-end housing buyers' utility rental benefits, w_Q. We note that high-floor high-end housing has luxury premium returns in three cities, and the luxury premium returns of high-end housing in Taipei and Kaohsiung are higher than those of Taichung. If builders want to invest in high-end housing, they can invest in Taipei and Kaohsiung, but they must carefully consider the middle-to-low floor buildings in Taichung, as their luxury premium returns are negative. According to the above analysis, builders must consider three important points: 1) the high-end housing buyers' utility rental benefit w_Q, 2) the income of the local residents, and 3) the cost of building.

Conclusions

Buyers of high-end, owner-occupied homes, often pay higher prices to buy high-end housing. They are motivated to consume highly conspicuous high-end housing and flaunt their wealth, thereby achieving better social status (Veblen 1899). High-end housing buyers are wealthy individuals who consume luxury high-end housing at a price higher than their intrinsic value. Builders can earn strictly positive profits when high-end housing is of Types A and C. Because certain types of homes are more visible in terms of size, location, and design, high-end housing buyers would purchase high-end housing not only for the pleasure of their intrinsic value but also for additional social esteem because such housing signals their own wealth, and the value of w_Q will increase. If high-end housing can create a high-value w_Q, the buyer will earn higher luxury premium returns.

According to our results, high-end housing can create higher luxury premium returns, but not all builders can earn higher luxury premium returns. Our results show that Types B and D are not sound investment projects, especially Type B. Builders who have little experience with high-end housing investment must prepare to investigate what high-end housing buyers enjoy.

Our results show that location, one of the features of housing, may be the most crucial factor. Type C can gain the highest profits with investment α at a low level because of homes with excellent neighbourhood amenities, the same race of neighbourhoods, and good transport facilities (Cutler et al., 1999; Crowder, 2000; Lee & Mori, 2016). If builders have an excellent location, they can construct high-end housing without investing too much α to gain higher luxury premium returns, as in Type A. Owing to the scarcity of land properties and the high costs of well-located land, builders with well-located land can construct high-end housing that is not easy to construct. If builders want to earn more profits, they can invest in luxury buildings to raise w_Q (e.g., they can build pleasant and spacious homes with excellent public amenities and designs that signal the wealth of buyers who want to display their own wealth and achieve greater social status), as in Type A.

Our model can appraise luxury valuations and premium returns in high-end housing. In contrast, Lee and Mori (2016) discovered through empirical research that conspicuous behaviour has a positive relationship with high-end housing premiums. Our results show that not all high-end housing units have positive premiums. If builders do not attract high-end housing buyers to invest in high-end housing, they will experience negative premium returns.

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Author contributions

Chih-Hsing Hung conceived the study and is responsible for the theoretical work. Shyh-Weir Tzang, the corresponding author, and the other two coauthors are responsible for the data analysis, numerical simulation and interpretation.

Disclosure statement

Authors do not have any competing financial, professional, or personal interests from other parties.

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Appendix A

Proof: By optimal selling price for $H_L^{\#}$, we obtain

$$H_L^{\#}=\frac{\left(\frac{(w_H-z)}{r}+\frac{w_H\Delta w_H}{r-\Delta w_H}\right)H_0\lambda_1-\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times\left(1-t_c\right)\lambda_1-\lambda_1H_0\times t_c}{2\times\left(1-t_c\right)\lambda_1-\beta\lambda_1-\left(1-t_c-\beta\right)}$$

Home equity is as follows:

$$E=F^H+F^{int}-F^k-F^D \quad \text{(A.1)}$$

Substituting Equations (7), (10), (12), and (14) into Equation (A.1) yields

$$E=F^H+F^{int}-F^k-F^D=\left[\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0-H\times\left(1-t_c\right)\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[H_L-\left(H_L-H_0\right)\times t_c\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}+\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-\frac{PMT}{r}\left(1-e^{-rT}\right)\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-M_0\times\left(\frac{H}{H_L}\right)^{\lambda_1}-\beta H_L\left(\frac{H}{H_L}\right)^{\lambda_1}. \quad \text{(A.2)}$$

We can arrange Equation (A.2) as:

$$E=\left[\begin{array}{l}\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0-H\times\left(1-t_c\right)+\\\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\end{array}\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[H_L-\left(H_L-H_0\right)\times t_c-M_0-\beta H_L\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}. \quad \text{(A.3)}$$

Differentiating Equation (A.3) with respect to H_L, a "smooth-pasting" condition (Leland, 1994) at $H = H_L$, and solving for optimal selling price for $H_L^{\#}$:

$$\left.\frac{\partial E}{\partial H_L}\right|_{H=H_L}=0$$

$$\left.\frac{\partial E}{\partial H_L}\right|_{H=H_L}=\left[\begin{array}{l}\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0-H\times\left(1-t_c\right)+\\\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\end{array}\right]\lambda_1\frac{1}{H_L}\times\left(\frac{H_L}{H_L}\right)^{\lambda_1}-\left[H_L-\left(H_L-H_0\right)\times t_c-M_0-\beta H_L\right]\lambda_1\frac{1}{H_L}\left(\frac{H_L}{H_L}\right)^{\lambda_1}+\left[\left(1-t_c\right)-\beta\right]\times\left(\frac{H_L}{H_L}\right)^{\lambda_1}=0. \quad \text{(A.4)}$$

We can arrange Equation (A.4) as follows:

$$\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0\lambda_1-\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times\left(1-t_c\right)\lambda_1-\lambda_1H_0\times t_c=\left[2\times\left(1-t_c\right)\lambda_1-\beta\lambda_1-\left(1-t_c-\beta\right)\right]H_L; \quad \text{(A.5)}$$

$$H_L^{\#}=\frac{\left(\frac{w_H}{r-\Delta w_H}-\frac{z}{r}\right)H_0\lambda_1-\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times\left(1-t_c\right)\lambda_1-\lambda_1H_0\times t_c}{2\times\left(1-t_c\right)\lambda_1-\beta\lambda_1-\left(1-t_c-\beta\right)}. \quad \text{(A.6)}$$

Appendix B

Proof: By optimal selling price for H_L^{*}, we obtain

$$H_L^{*}=\frac{\left[\begin{array}{l}\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\left(1-t_c\right)-\\H_0\left(1+\alpha\right)t_c+H_0\times\left(1+\gamma\right)\times R\end{array}\right]\lambda_1}{\left(1+\alpha\right)\left(1-t_c\right)\lambda_1+\left[\left(1+\alpha\right)\left(1-t_c\right)-\beta\right]\left(\lambda_1-1\right)}$$

By the home equity equation:

$$E=F^Q+F^{int}-F^k-F^D. \quad \text{(B.1)}$$

We can substitute Equations (19), (10), (12), and (14) into Equation (B.1) to obtain the following:

$$E=\left[\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[H_L(1+\alpha)-\left(H_L(1+\alpha)-H_0\right)t_c\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}+\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-\frac{PMT}{r}\left(1-e^{-rT}\right)\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-M_0\times\left(\frac{H}{H_L}\right)^{\lambda_1}-\beta H_L\left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{B.2}$$

By setting $E^u=E-CT$, we can get the following:

$$E^u=F^Q+F^{int}-F^k-F^D-F^{UR}=\left[\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[H_L(1+\alpha)-\left(H_L(1+\alpha)-H_0(1+\alpha)\right)t_c\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}+\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-\frac{PMT}{r}\left(1-e^{-rT}\right)\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-M_0\times\left(\frac{H}{H_L}\right)^{\lambda_1}-\beta H_L\left(\frac{H}{H_L}\right)^{\lambda_1}-\left[H_0\times(1+\gamma)\times R\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{B.3}$$

We can rearrange Equation (B.3) as follows:

$$E^u=\left[\begin{array}{l}\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)+\\ \left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\end{array}\right]\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[\begin{array}{l}H_L(1+\alpha)-\left(H_L(1+\alpha)-H_0(1+\alpha)\right)t_c-\\ M_0-\beta H_L-H_0\times(1+\gamma)\times R\end{array}\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{B.4}$$

Differentiating Equation (B.4) with respect to H_L, a "smooth-pasting" condition (Leland, 1994) at $H = H_L$, we can derive the optimal sell price for H_L^*:

$$\left.\frac{\partial E^u}{\partial H_L}\right|_{H=H_L}=\left[\begin{array}{l}\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H(1+\alpha)(1-t_c)+\\ \left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\end{array}\right]\times\frac{\lambda_1}{H_L}-\left[\begin{array}{l}H_L(1+\alpha)-\left(H_L(1+\alpha)-H_0(1+\alpha)\right)t_c-\\ M_0-\beta H_L-H_0\times(1+\gamma)\times R\end{array}\right]\times\frac{\lambda_1}{H_L}+\left[(1+\alpha)-(1+\alpha)t_c-\beta\right]=0. \tag{B.5}$$

We can further rearrange Equation (B.5) to obtain the following:

$$\left[\begin{array}{l}\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-H_L(1+\alpha)(1-t_c)+\\ \left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\end{array}\right]\lambda_1-\left[\begin{array}{l}H_L(1+\alpha)-\left(H_L(1+\alpha)-H_0(1+\alpha)\right)t_c-\\ M_0-\beta H_L-H_0\times(1+\gamma)\times R\end{array}\right]\lambda_1+\left[(1+\alpha)-(1+\alpha)t_c-\beta\right]H_L=0 \tag{B.6}$$

and

$$H_L(1+\alpha)(1-t_c)\lambda_1+H_L(1+\alpha)\lambda_1-H_L(1+\alpha)t_c\lambda_1-\beta H_L\lambda_1-\left[(1+\alpha)-(1+\alpha)t_c-\beta\right]H_L=\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0\lambda_1+\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)\lambda_1\times t_c-\frac{PMT}{r}\left(1-e^{-rT}\right)\lambda_1-H_0t_c(1+\alpha)\lambda_1+M_0\lambda_1+H_0\times(1+\gamma)\lambda_1\times R. \tag{B.7}$$

Therefore, we have:

$$H_L^*=\frac{\left[\begin{array}{l}\left(\frac{w_Q}{r-\Delta w_Q}-\frac{z}{r}\right)H_0-\left(\frac{PMT}{r}\left(1-e^{-rT}\right)-M_0\right)(1-t_c)-\\ H_0(1+\alpha)t_c+H_0\times(1+\gamma)\times R\end{array}\right]\lambda_1}{(1+\alpha)(1-t_c)\lambda_1+\left[(1+\alpha)(1-t_c)-\beta\right](\lambda_1-1)}. \tag{B.8}$$