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GEOMETRIC DILUTION OF PRECISION OF THE GNSS FOR MARS (GNSS FATIMA)

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Abstract. Positioning on Mars is one of the critical aspects of every planetary mission. Current complex planetary exploration systems (orbital and surface) rely on complex navigation and positioning systems, which make these systems complicated, expensive and their missions dangerous. The project of the global navigation satellite system for Mars (proposed system name – FATIMA) can make this and even future manned missions more safe, less expensive and the whole positioning in real time more reliable. The GNSS can be used by more systems or users simultaneously. In this research paper we focus on possible positioning errors when such a system is used. This research is focused on the GDOP – Geometric Dilution of Precision as one of the main factors influencing the GNSS.

Keywords: GNSS for Mars, GDOP in Mars conditions, project FATIMA, positioning on Mars, WGDOP.

1. Introduction

The determining of the exact position using the global navigation satellite systems is based on the principle of measuring the distance between the user's receiver and the visible navigation satellites of the system. This principle could be described as a positioning measurement of the point of the intersection of spheres the radius of which is given by the measured distances. This system is also called a distance measuring system. The measured value is the time period needed for the radio signal to travel from the satellite antenna to the antenna of the user's receiver. The propagation speed is equal to the speed of light. Each satellite in the navigation message in addition to other data also includes its orbital parameters (ephemeris), from which we can calculate the current position of the satellite. The time of space segment is fully synchronized by the pair of atomic clocks for each satellite, but the time base of the space segment and satellite receivers cannot be synchronized. This happens because it is not possible to install a time etalon of the corresponding accuracy in the receivers. This is the reason of the existence of an unknown shift of the time bases of space and the user's segments Δt or t. Because of this, the unknowns x, y, z and t of the user are present and the position of the user can be calculated using four simultaneous equations of the measured pseudo ranges. The total system error TSE[1] is the combination of the navigation system error NSE[2], the flight-technical error FTE[3], the system computation tolerance ST[4] and the display error (if it exists). The estimation of the navigation system error (NSE) can be expressed as the multiplication of the user equivalent range error (UERE[5]) and the coefficient characterizing the deployment of satellites in the hemisphere (DOP[6]). The DOP parameters act as the multiple parameters in relation to the calculation of the mean square error of measurement of a given parameter (Hrdina *et al.* 1996). The coefficients of the dilution of precision DOP can be described as:

- GDOP – geometric DOP (4D, affects the determination of all values – position in the space and also the corrections of the receiver's clocks).
- PDOP – position DOP (3D, affects the determination of the position in the space).
- HDOP – horizontal DOP (2D, affects the determination of the horizontal position in the space; is not very influenced by the change of latitude).
- VDOP – vertical DOP, affects the determination of altitude, its rate depends on the latitude (latitude minimum = the inclination of the orbit).
- TDOP – time DOP, affects the determination of corrections of the receiver's clocks.

In the computation process of precise positioning using the GNSS, it is necessary consider other factors which have a significant influence on the final determination of the position and accuracy. These undoubtedly include the effects of the ionosphere of Mars, the influence of the troposphere as well as the reflections from a different topography, where at that time the system is currently in use. In the case of terrain obstacles on Mars these will not be the obstacles similar to those on Earth (buildings). This will be for example the use of the system in the deep valleys of canyons. These influences can be continuously included in a real calculation. In this case, it is known as the calculation of the WGDOP[7]. The WGDOP value is a weighted geometric deviation of accuracy and is calculated differently from the GDOP. In the calculation of the geometric accuracy degradation it is thus possible to apply two kinds of calculations – the calculation of the GDOP and the calculation of the WGDOP. In this work we will focus on the theoretical calculation sequence of the GDOP errors when using the simulated GNSS[8] for Mars (project FATIMA). In the case of the GDOP, there will be a deterioration in the accuracy caused by the geometry of the constellation of satellites at the time of measurement. It is a mathematical function representing the quality of the positioning based on the mutual geometrical arrangement of the satellites and of the receiver. The lower is the value of this function, the more accurate is the measurement. Then the lowest possible dilution of precision and the highest possible

[1] Total System Error.
[2] Navigational System Error.
[3] Flight Technical Error.
[4] System computation Tolerance.
[5] User Equivalent Range Error.
[6] Dilution of Precision.
[7] WGDOP – Weighted Geometric Dilution of Precision.
[8] GNSS – Global Navigation Satellite System.

accuracy of the positioning can be achieved. (Rapant 2002) Other planetary influences (ionosphere, troposphere) and other errors in the determination of the precise position using GNSS in this procedure do not apply. In this process, we considered the ideal environment to meet the ideal laboratory conditions.

2. Hypothesis

The first idea of a global navigation satellite system for planet Mars was based on the requirements as presented by the NASA's Jet Propulsion Laboratory (Caltech)[9]. The required system was required to provide the positioning and navigation services and also it was meant that this system should be able to create some kind of communication network. K. O'Keefe, from the DGE of University of Calgary in Canada[10] was working on the proposal of such a concept. The prepared concept included the constellation of six satellites on the low orbit of Mars. According to computations followed by a simulation in simulated Martian conditions, this system was able to provide its positioning services for users in the equatorial and the Polar Regions of Mars, but with significant deficiencies in the mid-latitude locations. Instantaneous positioning was also very limited because of the low number of satellites in the proposed constellation. It meant that the user always needed to wait for a suitable geometry of the visible satellites to allow the computation of a precise position. The lack of satellites also did not allow error measurements. These results were caused mainly by the aforementioned lack of the redundant satellites directly visible by the user's receiver. Any error measurement could not be subsequently automatically corrected, mistakes could not be replaced by the corrections and the following positioning was inaccurate. The navigation services based on real-time instantaneous positioning were not able in all locations. The concept was processed but the technical design and construction was not done. A further work dealing with the proposal of a system which would be able to provide satellite navigation and positioning services on Mars was the theoretical work describing the design of a possible Martian satellite telecommunication and positioning network. This work also considered the use of a constellation of six satellites on their circular orbits at a height of 600 km with inclinations of 10°, 50° and 79°. These orbital parameters were chosen because of the best possible coverage of the equatorial areas on Mars. The reason for this was the planned robotic sample return mission expecting a delivery of samples of Martian soil back to Earth. This mission was not realized and the robotic research of the planet Mars was then focused on in-situ exploration of Mars via the new mission called Mars Science Laboratory – the rover Curiosity includes an advanced automatic chemical laboratory. (Chemistry & Mineralogy X-Ray Diffraction (NASA 2015)). This theoretical proposal was then processed as a model but was never realized. This concept also did not include the wider use of the global navigation system on Mars which would be able to cover the entire planet with its positioning services. Any complex concept of the GNSS for Mars was never realized. When proposing the global navigation system for Mars, it is necessary to carry out more experiments, calculations and a detailed research of the natural Martian conditions and their possible influence on the entire functionality of such a system. One of the inseparable parts of the research is, therefore, the calculation of the types of deterioration in the positioning accuracy on Mars. The already mentioned also include the deterioration in the accuracy of the geometric dilution of precision (GDOP), on which we focus in this work. The calculation of the GDOP for Mars can be done in two ways: 1) by using precise coordinates of the satellites of the system calculated using a simulation model; or 2) using the known ephemeris of any of the terrestrial GNSS systems that we apply to the conditions of Mars. In the above mentioned cases the use of both methods was considered. In our research we focus on a detailed mathematical description of the theoretical method. We will use the simulated satellites on the Martian orbits with high altitudes similar to the GNSS systems used on Earth. Therefore, on Mars we will differ by an essential fact from the previous procedures and research of the GDOP which is the altitude of the orbits of the navigation satellites – in the previous cases orbits with altitudes at a 600–700 km maximum were used. (O'Keefe *et al.* 2005) When higher altitudes of the orbits are applied we expect a much higher effectivity of the whole GNSS system. We believe that on Mars it is technically possible to implement a functioning global navigation satellite system called FATIMA. This system will consist of three segments – space, command (control) and user segment. The space segment consists of satellites located on the defined orbits around Mars with stable altitudes[11], the control segment consists of an automatic re-translation station located on the moon of Mars – Phobos, and the command centre – on Earth. This retranslation station collects the data broadcast by the individual visible satellites in a non-stop operation and will send a package of these data to Earth in a two phase interval[12]. The con-

[9] Caltech – California Institute of Technology, USA.

[10] Department of Geomatics Engineering, University of Calgary, CA.

[11] The heights (altitudes) of the orbits were defined according to the results from the analysis of the vertical ionospheric profiles of day side a night side of Mars.

[12] Phobos orbits Mars twice per day. Its orbital period is 7 hrs and 39 min. According this it can be in suitable position for broadcasting the data to Earth twice per 24 hrs.

trol centre monitoring the functionality and operation of the entire GNSS system on Mars is based on Earth. This centre is able to manage any potential outage of the system. Of course the known time delay in the signal send/receive according to the actual distance between Mars and Earth must be considered. The user segment consists of the receivers on the surface of Mars, or in its atmosphere, or, eventually, on some of the lower orbits of Mars. The main target of this research is to answer the questions, or better to say to confirm or deny the hypotheses saying that the proposed GNSS FATIMA with the proposed parameters is able to provide its positioning services on Mars with acceptable errors. These errors cannot reach the critical values, because they guarantee the safe use of this system on Mars.

3. Cartographic constants on Mars

Precise maps of the Martian surface based on precise measurements and created on the basis of unified, internationally recognized constants and standards are the basic condition for any precise scientific research of such large areas and regions on Mars. This is necessary for even small areas. Similar to Earth, the cartography of Mars comes from three basic parameters – the prime meridian, the equator and the zero altitude. Other important constants are the rotation period of the planet, the angle of the axis of planet's rotation and the orientation in the epoch. The parameters describing Mars' rotation were derived from the values which were determined on the basis of the tracking[13] of the Mars Pathfinder and Viking 1 and Viking 2 landers. (Duxbury *et al.* 2002) The right ascension[14] α and declination δ in the given time *t* are expressed as:

$$\alpha = \frac{317.68143° - 0.1061°}{\text{century} \cdot \mathrm{T}};$$
$$\delta = \frac{52.88650° - 0.0609°}{\text{century} \cdot \mathrm{T}}, \tag{1}$$

where *T* is the number of Julian centuries from the time *t* from the standard epoch J2000.0. (Folkner *et al.* 1997) The zero altitude on Mars is determined according to the measurements of the Mars Orbiter Laser Altimeter experiment (MOLA)[15] carried out in 2001. The zero altitude was calculated as the average value which is equal to the middle radius of the planet on its equator.

[13] Tracking–projection of the trajectory of some surface probe (rover) or orbital probe on the reference surface of the planet (Mars).

[14] Right ascension – the angular distance measured eastwards along the celestial equator from the vernal equinox to the hour circle of the point in question.

[15] MOLA (Mars Orbiter Laser Altimeter) is the experiment of Mars Global Surveyor (NASA).

Therefore, the gravitational and rotational value of the equipotential surface on the equator in this middle radius of Mars was used. (Smith *et al.* 2001) Another important cartographic constant of Mars is the prime meridian. This is determined as the meridian crossing the crater *Airy-0*. This crater is located in the area of the Sinus Meridiani and its size is 0.5 km in diameter (middle diameter). The equator is determined as the zero parallel projected on the surface of the rotational ellipsoid by the plane crossing the exact center of the ellipsoid. The most suitable planetary parameters of the rotational ellipsoid for Mars are:

$$A = 3396.19 \text{ km};$$
$$B = 3376.20 \text{ km}.$$

The *A* value represents the middle equatorial axis and the *B* value represents the middle polar axis of the ellipsoid. The most suitable center of the ellipsoid with a minimal deviation from the center of the planetary body is calculated according the gravitational force. The exact center was calculated as the center of the ellipsoid with a radius *R* = 3389.50±0.2 km. (Duxbury *et al.* 2002). In the project of the satellite navigation system for Mars, we will use the planetocentric coordinate system. This system uses the positive coordinates in the direction from the west to the east. We will not use the planetographic coordinate system of Mars, because this is currently also considered as a standard model. The planetocentric system will be more efficient for the Mars GNSS system. (Kozar 2015) In the calculation formulas and definitions we will focus on the Cartesian coordinate system using coordinates *X, Y, Z*. The example of the coordinate system is explained in Figure 1 below.

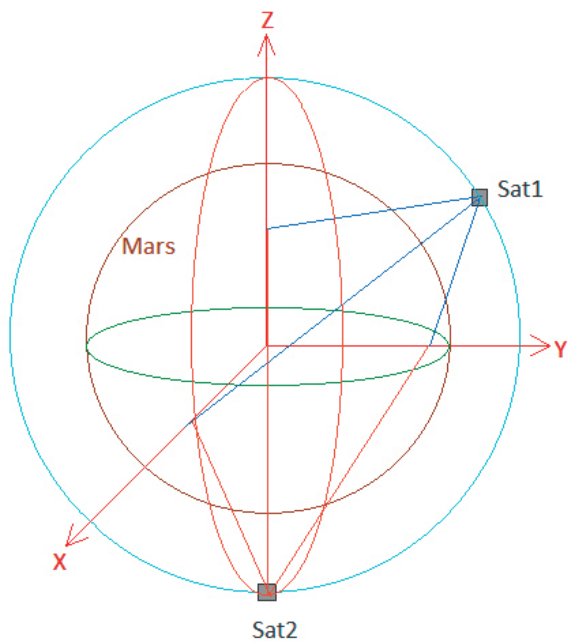

Fig. 1. Coordinate system with the example of two satellites. Image credit: Jozef Kozar, Faculty of Aeronautics of Technical University of Kosice

4. Experiment

In this phase of research we will focus on the continuity of mathematical calculation and on the analysis of the outputs. We will use the basic formulas for the computation of the GDOP for the GNSS. (Subirana *et al.* 2015) In the process of computation of the precise position of the receiver located on the Martian surface with the satellite navigation system for Mars, it is also suitable to consider the errors which are continuously originating for example from the change of the receiver's location on the planet. The geometric dilution of precision would mean in this case a continuing increase of inaccuracies and the need for their corrections with the help of an integrated calculation. In this study, however, we focus on the deterioration of the calculation of the geometric dilution of precision in an ideal environment – the position of the user/receiver on the Martian surface will be static. In this research we will not consider the state when the user/receiver is moving on the surface or in low altitudes. The principle of the geometric dilution of precision is based on the assumption of the influence that the measurement errors have on the final determination of position according to the geographic coordinates. This principle was described by Dudek and Jenkin (Dudek, Jenkin 2000) in formula (2).

$$GDOP = \frac{\Delta\left(\text{calculated position } x,\, y,\, z\right)}{\Delta\left(\text{measured data } x_n,\, y_n,\, z_n,\, t_n\right)}, \tag{2}$$

where x, y, z are the coordinates calculated by the receiver based on the measured data x_n, y_n, z_n and the time t_n. Even before the experimental computation, it the values of the dilution of precision need to be defined according to the standards defined in the Table 1.

Due to the fact that the GNSS will use the planetocentric coordinate system of Mars, it is possible to describe the relation between the user and the satellite with the following formula:

$$r_i = \sqrt{\left(x - X_i\right)^2 + \left(y - Y_i\right)^2 + \left(z - Z_i\right)^2} + c \cdot t_b + v_{ri}, \tag{3}$$

where (x, y, z) are the coordinates of the user/receiver and (X_i, Y_i, Z_i) are the coordinates of the satellite. The value c is a constant which defines the speed of light and t_b defines the deviation of the time bases of the space and user segments. The value v_{ri} defines the noise of the measured pseudorange (Chen *et al.* 2013). In the computation it is necessary to define the formal precision by the determination of values P_{xx}, P_{yy}, P_{zz}, P_{tt}. These values will be defined for each element of the matrix as:

$$P_{\Delta X},$$

which expresses the relation between the values of the geometric matrix G, the transformation matrix R and the matrix of the time dilution of precision T:

$$P = \left(G^T \cdot R^{-1} \cdot G\right)^{-1}. \tag{4}$$

From the described relation, it is possible to express the values of the standard deviations for the individual coordinates:

$$\sigma_X = \sqrt{P_{XX}},\ \sigma_Y = \sqrt{P_{YY}},\ \sigma_Z = \sqrt{P_{ZZ}},\ \sigma_T = \sqrt{P_{TT}}. \tag{5}$$

The above mentioned formulas define the characteristics of the quality of the coordinates and the determined time. These do not determinate the error, but they express the uncertainty in the error estimates in the formal accuracy. Previous equations defined the errors in coordinates X Y Z. Despite this, it is necessary to think of the horizontal error, the vertical error, or,

Table 1. Description of the values of the geometric dilution of precision and the influence on precise positioning (Langley 1999)

Value of DOP	Evaluation	Description
<1	ideal	The best possible determination of the position with minimum errors.
1–2	excellent	The determination of position is precise and meets the conditions required by most of the applications where needed.
2–5	good	The calculated results meet the conditions for positioning. It is still possible to use the system for navigation.
5–10	average	The measurements of position can be used in computations but the improvement of their quality is necessary.
10–20	poor	The calculated position is suitable only for use as a rough estimate of the actual position.
>20	very poor	Calculations are very inaccurate and should not be used in any application.

simply, about the "eastern" error (E[16]), "northern" error (N[17]) and "vertical" error (U[18]). The following matrix *R* defines the relations among the individual horizontal coordinates λ (eastern longitude) and φ (northern latitude)[19] (Subirana *et al.* 2015):

$$R = \begin{bmatrix} -\sin\lambda & -\sin\varphi\cos\lambda & \cos\varphi\cos\lambda \\ \cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi\sin\lambda \\ 0 & \cos\varphi & \sin\varphi \end{bmatrix}. \quad (6)$$

Going back to the definition and the expression of relation between the satellite and the user, equation (2) is linearized through the Taylor series. This can be expressed as:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}.(a)}{n!}.(x-a)^n, \quad (7)$$

where

$$f^{(n)}.(a) \quad (8)$$

– is the *n*-th derivation of function *f* in point *a;*
– *f* has around the point *a* derivations of all series (Hazewinkel 2001).

The equation expressing distance between the user and the satellite after the adjustment is as follows:

$$\Delta r = r_i - \hat{r}_i \cong e_{i1}\delta_x + e_{i2}\delta_y + e_{i3}\delta_z + C \cdot t_b + v_{ri}, \quad (9)$$

where δ_x, δ_y and δ_z are the respective compensations of the coordinates *x*, *y* and *z:*

$$e_{i1} = \frac{\hat{x} - X_i}{\hat{r}_i}, \; e_{i2} = \frac{\hat{y} - Y_i}{\hat{r}_i}, \; e_{i3} = \frac{\hat{z} - Z_i}{\hat{r}_i}; \quad (10)$$

$$\hat{r}_i = \sqrt{(\hat{x} - X_i)^2 + (\hat{y} - Y_i)^2 + (\hat{z} - Z_i)^2}. \quad (11)$$

Values (*ei1, ei2, ei3*), *i* = 1, 2... *n*, represent the vector of direct visibility (VDV) in the direction from the satellite to the user/receiver. The linear equations of measurement of the pseudoranges have the following expression:

$$z = H\delta + v, \quad (12)$$

where

$$z = \begin{bmatrix} r_1 - \hat{r}_1 \\ r_2 - \hat{r}_2 \\ . \\ . \\ . \\ r_n - \hat{r}_n \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ c \cdot t_b \end{bmatrix}, \quad v = \begin{bmatrix} v_{r1} \\ v_{r2} \\ . \\ . \\ . \\ v_{rn} \end{bmatrix},$$

$$H = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 1 \\ e_{21} & e_{22} & e_{23} & 1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ e_{n1} & e_{n2} & e_{n3} & 1 \end{bmatrix}. \quad (13)$$

As mentioned, the last expression of value *H* represents the geometric matrix (Chen *et al.* 2013). According to the lowest exponentiation of the algorithm of direct visibility, the solution of equation (12) can be expressed as

$$\hat{\delta} = \left(H^T \cdot H\right)^{-1} \cdot H_z \cdot \quad (14)$$

According to the error in the determination of the pseudoranges, these are not correlated by the same divergences σ^2; the covariance error matrix can be expressed as:

$$E\left[\left(\hat{\delta} - \delta\right)\left(\hat{\delta} - \delta\right)^T\right] = \sigma^2 . \left(H^T H\right)^{-1}. \quad (15)$$

The divergences are the functions of the diagonal elements $(H^T H)^{-1}$. The geometric dilution of precision (*GDOP*) is a degree of precision of the positioning system, so it clearly depends on the geometric matrix *H*:

$$GDOP = \sqrt{tr\left(H^T.H\right)^{-1}}. \quad (16)$$

In reality each measurement error does not express the same divergence. The covariance matrix expresses this inaccuracy in the measurement of the pseudoranges. This matrix can be written as follows:

$$E\left(vv^T\right) = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}. \quad (17)$$

5. Calculation and experimental results

Before the experiment, it is necessary to determine the coordinates and the time (T) of a minimum of four directly visible satellites of the system (satellites *A, B, C, D*). The experimental values of the coordinates of the satellites *X, Y, Z* in the coordinate system MCMF[20] have been calculated using the conversion from the coordinates in the source coordinate system LLH[21]. For this the real coordinates of the four simulated satellites of the satellite navigation system on Mars (system FATIMA)

[16] E – abbreviated from "east".

[17] N – abbreviated from "north".

[18] U – abbreviated from "up" (in vertical direction).

[19] In case of Mars we consider the zero meridian defined by the line which begins in the North Pole, crossing the crater Airy-0 and ending in the South Pole.

[20] MCMF – Mars Centered, Mars Fixed.

[21] LLH – Latitude, Longitude, Height.

have been used. The ephemeris of these satellites have been verified in the simulation programs NASA MarsTrek[22] and JsatTrak[23] modified for the conditions of Mars. We will use the following values for the coordinate system MCMF:

$A_{x1} = -2102.102$ $Bx_2 = -4483.582$
$A_{y1} = 12521.389$ $By_2 = 14692.645$
$Az1 = 9514.14$ $Bz_2 = -3993.467$

$C_{x3} = -12047.02$ $Dx_4 = -12208$
$C_{y3} = 1227.94$ $Dy_4 = 8931.634$
$C_{z3} = 10249.154$ $Dz_4 = -4805.775$

As the position of the receiver on Mars the real position of the rover Curiosity[24] of the mission MSL[25] of NASA was used. The given position of the receiver (R) in the coordinate system MCMF for the values X, Y, Z is as follows:

$Rx = -7171.77$
$Ry = 6585.154$
$Rz = -778.164$

For each satellite from the mentioned four satellites $i = 0\ ..\ 4$.

The standard divergence r_1, r_2, r_3, r_4 of the individual coordinates is obtained from formula (11):

$$r_1 = \sqrt{\left(A_{x1} - R_x\right)^2 + \left(A_{y1} - R_y\right)^2 + \left(A_{z1} - R_z\right)^2}\ ;$$

$$r_2 = \sqrt{\left(B_{x2} - R_x\right)^2 + \left(B_{y2} - R_y\right)^2 + \left(B_{z2} - R_z\right)^2}\ ;$$

$$r_3 = \sqrt{\left(C_{x3} - R_x\right)^2 + \left(C_{y3} - R_y\right)^2 + \left(C_{z3} - R_z\right)^2}\ ;$$

$$r_4 = \sqrt{\left(D_{x4} - R_x\right)^2 + \left(D_{y4} - R_y\right)^2 + \left(D_{z4} - R_z\right)^2}\ .$$

Following this, we can calculate the vectors of direct visibility e_i from formula (10):

$$e_{x1} = \frac{A_{x1} - R_x}{r_1};\ e_{y1} = \frac{A_{y1} - R_y}{r_1};\ e_{z1} = \frac{A_{z1} - R_z}{r_1};$$

$$e_{x2} = \frac{B_{x2} - R_x}{r_2};\ e_{y2} = \frac{B_{y2} - R_y}{r_2};\ e_{z2} = \frac{B_{z2} - R_z}{r_2};$$

$$e_{x3} = \frac{C_{x3} - R_x}{r_3};\ e_{y3} = \frac{C_{y3} - R_y}{r_3};\ e_{z3} = \frac{C_{z3} - R_z}{r_3};$$

$$e_{x4} = \frac{D_{x4} - R_x}{r_4};\ e_{y4} = \frac{D_{y4} - R_y}{r_4};\ e_{z4} = \frac{D_{z4} - R_z}{r_4};$$

[22] NASA Jet Propulsion Laboratory Mars Trek Simulation.

[23] JSatTrak is a Satellite tracking program written in Java. It allows to predict the position of any satellite in real time or in the past or future. It uses advanced SGP4/SDP4 algorithms developed by NASA/NORAD.

[24] Gale crater, Latitude: 4°35´22,2´´ S; Longitude: 137°26´30,12´´ E.

[25] Mars Science Laboratory.

The time is marked as D_{t1}, D_{t2}, D_{t3}, D_{t4}. Following the use of the formula of the matrix H in relation (13):

$$H = \begin{bmatrix} e_{x1}\, e_{y1}\, e_{z1}\, D_{t1} \\ e_{x2}\, e_{y2}\, e_{z2}\, D_{t2} \\ e_{x3}\, e_{y3}\, e_{z3}\, D_{t3} \\ e_{x4}\, e_{y4}\, e_{z4}\, D_{t4} \end{bmatrix}.$$

After the use of formula (14), following the calculation and the final modification, the values of the coordinate compensations will be obtained. After the implementation of these values in the formula (16) the result is the value of the GDOP = 2.37668566431.

6. Conclusions

For the determination of the precise position of the user on the surface of Mars with the satellite navigation system, the visibility of at least four satellites is needed. In the case of the proposed satellite navigation system FATIMA, it is necessary to use a configuration based on the known parameters applied to the current global navigation satellite systems on Earth. According to the results of the analysis of the vertical ionospheric profiles of Mars, we think that the ionosphere of Mars will not have a significant influence on the broadcasting of the navigation signal. However, the ionosphere will cause some errors in the computation process because of the time delay caused by the ionosphere refraction. Also we think that the atmospheric conditions and the physical influences of the Martian environment will not cause any significant obstacles in positioning using the proposed system. These planetary influences are of course not consider as insignificant and we will include them in the form of statistical discrepancy.[26] In this research we were focused on the calculation of the coefficient of the geometric dilution of precision (GDOP) of a simulated user on Mars using the simulated global navigation satellite system for Mars (GNSS FATIMA). After the application of the coordinates of the simulated satellites in the given time epoch, we received the calculated coefficient of the GDOP = 2.37668566431 and, according the following analysis of this result, we claim that the evaluation of the measurement and the calculation of the precise position on the surface of Mars is good. The positioning measurements can be used in the calculations. A further possible improvement of the quality of the positioning can be considered. A more precise positioning can be achieved, for example, with the use of some kind of differential reference station, which would be located in the area where the global navigation system will be directly applied and used for scientific research.

[26] The statistical error will be defined at the end of the entire research of the Theoretical Concept of Satellite Navigation System for Mars (GNSS FATIMA).

In our experiment we have used the direct visibility of four satellites. From the calculated value of the GDOP coefficient we can see that the mutual geometric position of the satellites at the time of the measurements was not the best one. In the real process of positioning with the GNSS in real conditions we recommend to achieve the best possible visibility of more satellites at the same time. The geometry of the satellites used for the positioning has a significant influence on the precision of the positioning. When satellites are located in a relatively small area, then, according to our research, the entire system provides worse results than in a situation when the satellites are located in a wider area on the user's visible sky. The DOP parameters act as the multiple parameters in relation to the calculation of the mean square error of measurement of a given parameter. Therefore, to achieve a high positioning accuracy, it is necessary to achieve not only small values of the error caused by the apparent distance measurement, but also to use the largest number of visible satellites that are separated by the required distance.

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